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SIMPLIFIED CURVE FITS FOR THE THERMODYNAMIC PROPERTIES OF EQUILIBRIUM AIR

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SUMMARY

New improved curve fits for the thermodynamic properties of equilibrium air have been developed. The curve fits are for $p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(e,\rho)$, $s = s(e,\rho)$, $T = T(p,\rho)$, $h = h(p,\rho)$, $\rho = \rho(p,s)$, $e = e(p,s)$ and $a = a(p,s)$. These curve fits can be readily incorporated into new or existing CFD codes if "real-gas effects" are desired. The curve fits were constructed using Grabau-type transition functions to model the thermodynamic surfaces in a piecewise manner. The accuracies and continuity of these curve fits are substantially improved over those of previous curve fits appearing in NASA CR-2470. These improvements were due to the incorporation of a small number of additional terms in the approximating polynomials and careful choices of the transition functions. The ranges of validity of the new curve fits are temperatures up to 25,000 K and densities from 10^{-7} to 10^{3} amagats (ρ/ρ_0).

SYMBOLS

a	speed of sound
a_0	standard speed of sound = 331.36 m/s
e	specific internal energy, m^2/s^2
e_0	standard specific internal energy = 78,408.4 m^2/s^2
h	specific enthalpy, m^2/s^2
p	pressure, N/m^2
p_0	standard pressure = 101,330.0 N/m^2
R	gas constant = 287.06 m^2/s^2 - K
s	specific entropy, m^2/s^2 - K
s_0	standard specific entropy = 6779.2 m^2/s^2 - K
T	temperature, K
T_0	standard temperature = 273.15 K
$\tilde{\gamma}$	h/e
ρ	density, kg/m^3
ρ_0	standard density = 1.292 kg/m^3

Subscripts:

0	standard conditions

INTRODUCTION

Under subsonic flight conditions, air may be treated as an ideal gas composed of rigid rotating diatomic molecules, and the thermodynamic properties of such a gas are well known. However, under supersonic flight conditions, air may be raised to temperatures where the molecules can no longer be treated as rigid rotators. Thus, there is a very real need for the thermodynamic and transport properties of equilibrium air for the computation of flowfields around bodies in high-speed flight.

The equilibrium thermodynamic properties of air were calculated with good confidence as early as 1950. The earliest approach to compiling these properties was to present the information in the form of tables. Using this approach, Gilmore (ref. 1) and later, Hilsenrath and Beckett (ref. 2), computed the chemical composition, internal energy, entropy, compressibility and pressure of air as functions of temperature and density. The calculations in references 1 and 2 made use of multiple interation procedures. Hansen (ref.3) computed the compressibility, enthalpy, entropy, specific heats, speed of sound, Prandtl number, and coefficients of viscosity and thermal conductivity as functions of temperature and density in closed form and presented them in a tabular fashion. In addition, the thermodynamic properties of equilibrium air were compiled in the form of charts by Moeckel and Weston (ref. 4).

Subsequently, equilibrium air thermodynamic properties became available in the form of FORTRAN computer programs. These programs can be broadly divided into two categories. The first category consists of programs that compute the equilibrium composition and

thermodynamic properties using a harmonic-oscillator rigid-rotator model for the various component species of the gaseous mixture. Bailey (ref. 5) developed computer programs which used the temperature, density and molar concentrations of the various constituent species to calculate the pressure, gas constant, enthalpy, entropy, specific heats and coefficient of thermal conductivity. He presented these properties for a nine-species model as well as a eleven-species model of equilibrium air. Zeleznik and Gordon (ref. 6) developed a sophisticated computer program, improved later by Gordon and McBride (ref. 7), which computed the chemical equilibrium composition of complex chemical systems given the constituent species and one of five possible pairs of thermodynamic state combinations. Miner et al. (ref. 8) developed a 27-reaction equilibrium air program to generate the thermodynamic properties in tabular form as functions of pressure and temperature.

The second category of computer programs, which includes the present work, consists of programs that determine the thermodynamic properties of equilibrium air in a non-iterative fashion, using either interpolation or polynomial approximation techniques. Typically, the sources of data for these programs are references 1 to 4. Lomax and Inouye (ref. 9) developed FORTRAN programs to determine the speed of sound, enthalpy, temperature and entropy as functions of, either pressure and density, or pressure and entropy. Their programs used a nine point spline interpolation and required a lookup of over 10,000 tabulated values. The programs developed at NASA Ames Research Center in references 5 and 9 eventually evolved into the NASA RGAS program. The NASA RGAS program employs a cubic interpolation tech-

nique, with the associated table look-up of cubic coefficients, to compute the enthalpy, temperature, entropy and speed of sound of 13 different gas mixtures, including equilibrium air as functions of either pressure and density, or pressure and entropy. This program was an improvement over other sources of thermodynamic properties in this category in terms of accuracy and range of validity. For this reason it is still widely used. The NASA RGAS program was modified by Tannehill and Mohling (ref. 10) to allow internal energy and density to be used as independent variables for "time dependent" flow calculations. The major shortcoming of the RGAS program is that the table look-up of coefficients for the cubic interpolation makes it too cumbersome and time-consuming to be efficiently employed on an advanced computer.

Among the first to develop programs which approximated the thermodynamic properties as self-contained closed-form expressions was Grabau (ref. 11). He outlined a systematic technique of modelling the thermodynamic properties by polynomial expressions with exponential transitions. Using this technique he determined the enthalpy, entropy, speed of sound and compressiblity of equilibrium air as functions of pressure and density in the form of closed-form expressions (curve fits). Using Grabau's technique, Lewis and Burgess (ref. 12) obtained empirical equations for the density, enthalpy, speed of sound and compressibility factor of air as functions of pressure and entropy. However, these curve fits had a range of validity only up to 15,000 K and a pressure range of one-tenth to one atmosphere. The method of reference 11 was also employed by Barnwell (ref. 13) to curve fit $\tilde{\gamma}$ as a function of internal energy

and density, and temperature as a function of pressure and density for equilibrium air. Viegas and Howe (ref. 14) developed programs for the density, temperature, viscosity and Prandtl number of equilibrium air as functions of pressure and enthalpy in the form of curve fits with polynomials of up to degree 8 using least squares and Chebyshev polynomial fitting. Tannehill and associates (refs. 10, 15 and 16) developed simplified curve fits for the thermodynamic and transport properties of equilibrium air with the same range of validity as the NASA RGAS program. These curve fits include pressure, temperature, speed of sound and the coefficients of viscosity and thermal conductivity as functions of internal energy and density, and temperature and enthalpy as functions of pressure and density. The curve fits were constructed using Grabau-type transition functions in a manner similar to that of reference 11. In forming these curve fits, as many as five Grabau-type transition functions were joined with the perfect gas equation of state.

The references cited above are representative of the various approaches to obtaining the thermodynamic properties of equilibrium air and the list is by no means exhaustive.

One of the major shortcomings of the curve fits of references 10, 15 and 16 is the lack of continuity across the boundaries between the transition functions. As a consequence, numerical difficulties were sometimes encountered when these curve fits were employed in iterative flowfield computations. The primary objective of the present research effort was to alleviate this difficulty. At the same time, an attempt was made to improve the accuracy of the curve fits by incorporating a small number of additional terms which would not significantly increase the computation time.

Using an approach of carefully choosing the Grabau-type transition functions and using complete bicubic polynomials, the following curve fits were developed:

$p = p(e,\rho)$

$a = a(e,\rho)$

$T = T(e,\rho)$

$s = s(e,\rho)$

$T = T(p,\rho)$

$h = h(p,\rho)$

$\rho = \rho(p,s)$

$e = e(p,s)$

$a = a(p,s)$

These curve fits were based on the NASA RGAS data and have the same ranges of validity, namely, temperatures up to 25,000 K and densities from 10^{-7} to 10^{3} amagats (ρ/ρ_0).

BEHAVIOR OF AIR AT HIGH TEMPERATURE

When a gas composed of polyatomic molecules is heated to high temperatures, its composition changes as a result of the chemical reactions which take place. Such a situation exists behind the shock wave which envelops a vehicle reentering the atmosphere of the earth. As a result of the change in chemical composition, the thermodynamic properties of the gas also change. When the temperature of the gas is raised appreciably higher than the temperature at which dissociation reactions begin to occur, the electrons receive energy quanta because of the collisions between atoms. If the temperature, and hence the kinetic energy of the atoms is high enough

behavior provides a qualitative insight into the choice of the approximating functions. Figure 1 shows the function $\tilde{\gamma}$ plotted with respect to $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0)$ at a density of 10^{-7} amagats. Also shown are the various segments into which the curve may be divided, as indicated by A, AA, B, C and D. These segments are basically quadratic or linear curves which are joined together by transition

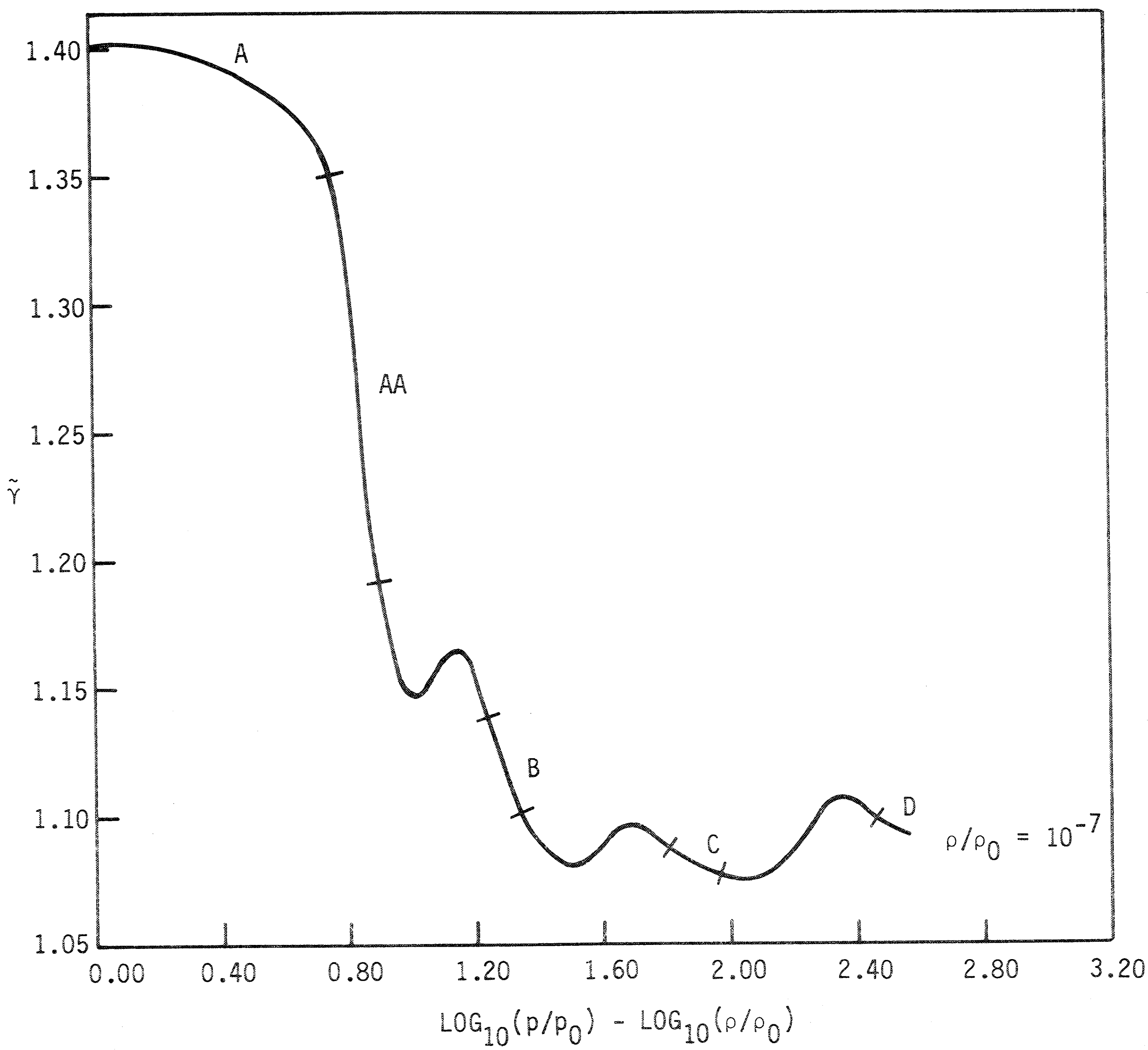

Figure 1. Variation of $\tilde{\gamma}$ with $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0)$

so that electrons are removed from their orbits, ionization of the gas takes place. The effects of dissociation and ionization of the gas on its thermodynamic properties are often referred to as "real-gas effects."

At room temperature, air consists of about 78 per cent diatomic nitrogen, 21 per cent diatomic oxygen and about 1 per cent of argon and traces of carbon dioxide. When the temperature of air is raised above room temperature, deviations from perfect gas behavior occur, i.e. the vibrational mode of the molecules becomes excited, dissociation of both oxygen and nitrogen molecules occur (although at different temperatures), nitric oxide is formed, etc. The chemical composition of air for densities lying between 10^{-2} and 10 times normal air density is approximately divisible into the following regimes.

1. $T < 2{,}500$ K. The chemical composition is substantially that at room temperature.
2. $2{,}500 < T < 4{,}000$ K. The oxygen dissociation regime; no significant nitrogen dissociation; slight NO formation.
3. $4{,}000 < T < 8{,}000$ K. The nitrogen dissociation regime; oxygen fully dissociated.
4. $T > 8{,}000$ K. Ionization of the atomic constituents.

In the flow calculations of air in thermodynamic equilibrium, it becomes important to know the various thermodynamic properties as functions of a pair of independent state variables. In order to illustrate the spatial behavior of these thermodynamic surfaces, a typical curve is examined here in some detail. The nature of the thermodynamic surface, with the plausible reasons for its undulating

curves. Two types of transition curves appear in figure 1 and these are illustrated in figures 2 and 3. Figure 2 shows a transition function which passes through a point of inflection and is referred

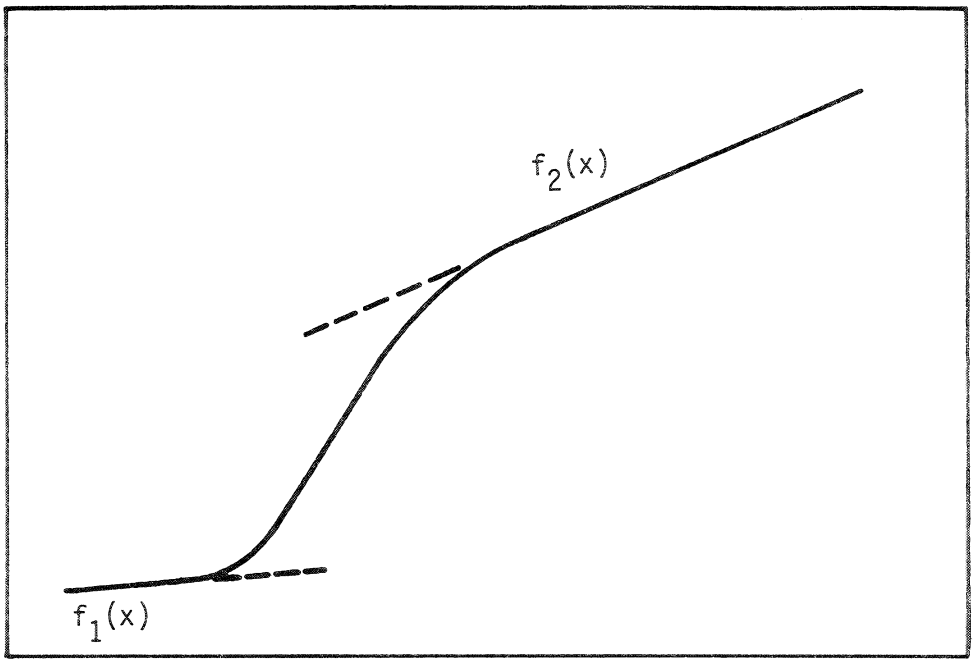

Figure 2. A transition with a point of inflection

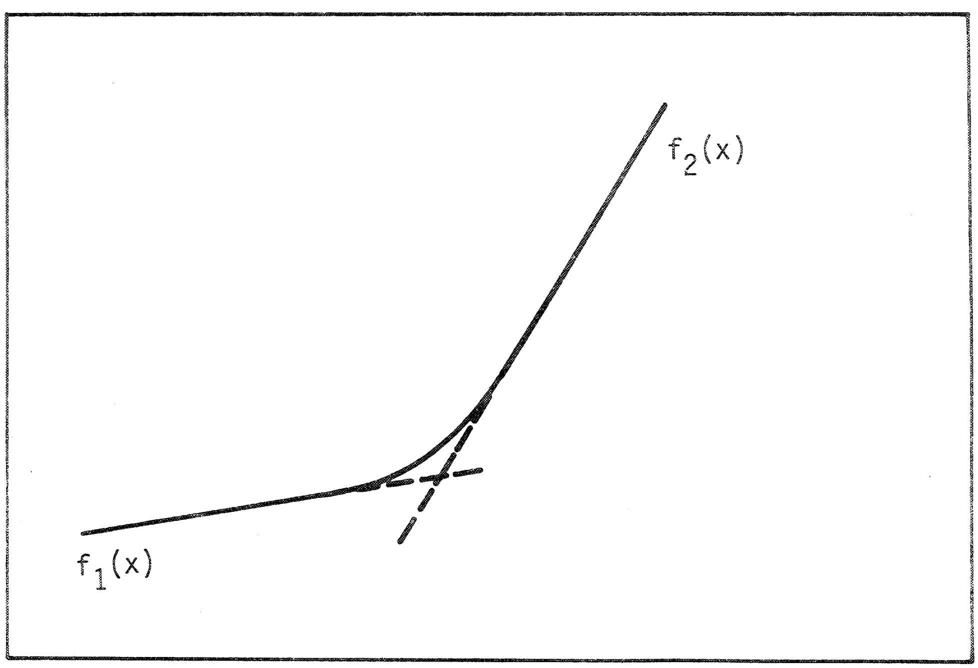

Figure 3. A transition without inflection

to as a transition with inflection. Figure 3 illustrates the second type of transition which is one without a point of inflection. It can be seen from figure 1 that $\tilde{\gamma}$ goes through three distinct transitions with inflections. According to reference 3 there is a definite correlation between these three transitions and the change in chemical composition of the air as the temperature increases: the first transition from AA to B is due to the oxygen dissociation reaction the second from B to C is due to the nitrogen dissociation and the third from C to D is due to the ionization reactions.

In addition to the three transitions with inflections in figure 1, there appears to be a relatively insignificant transition without an inflection between the curves A and AA. Also, after a careful examination of the segment D, it appears that it may actually be part of an incomplete transition with a point of inflection.

$\tilde{\gamma}$ is plotted as a function of $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0)$ for various densities in figure 4. As the density increases, it can be observed that pieces of the curve near C and D disappear one after the other until only a part of the transition into C remains at 10^3 amagats. The reason for this is that the compressibility factor decreases steadily as the density is increased isothermally. Hence, it also follows that isothermal points move rapidly along the curve from D toward C as the density is increased. Figure 4 provides an idea of the complexity of the problem of devising a practical method of modelling the collapse of the lower segments with increasing density. There appears to be a tendency for transitions with inflections to convert to transitions without inflections as the density increases. Reference 1 suggests that this conversion

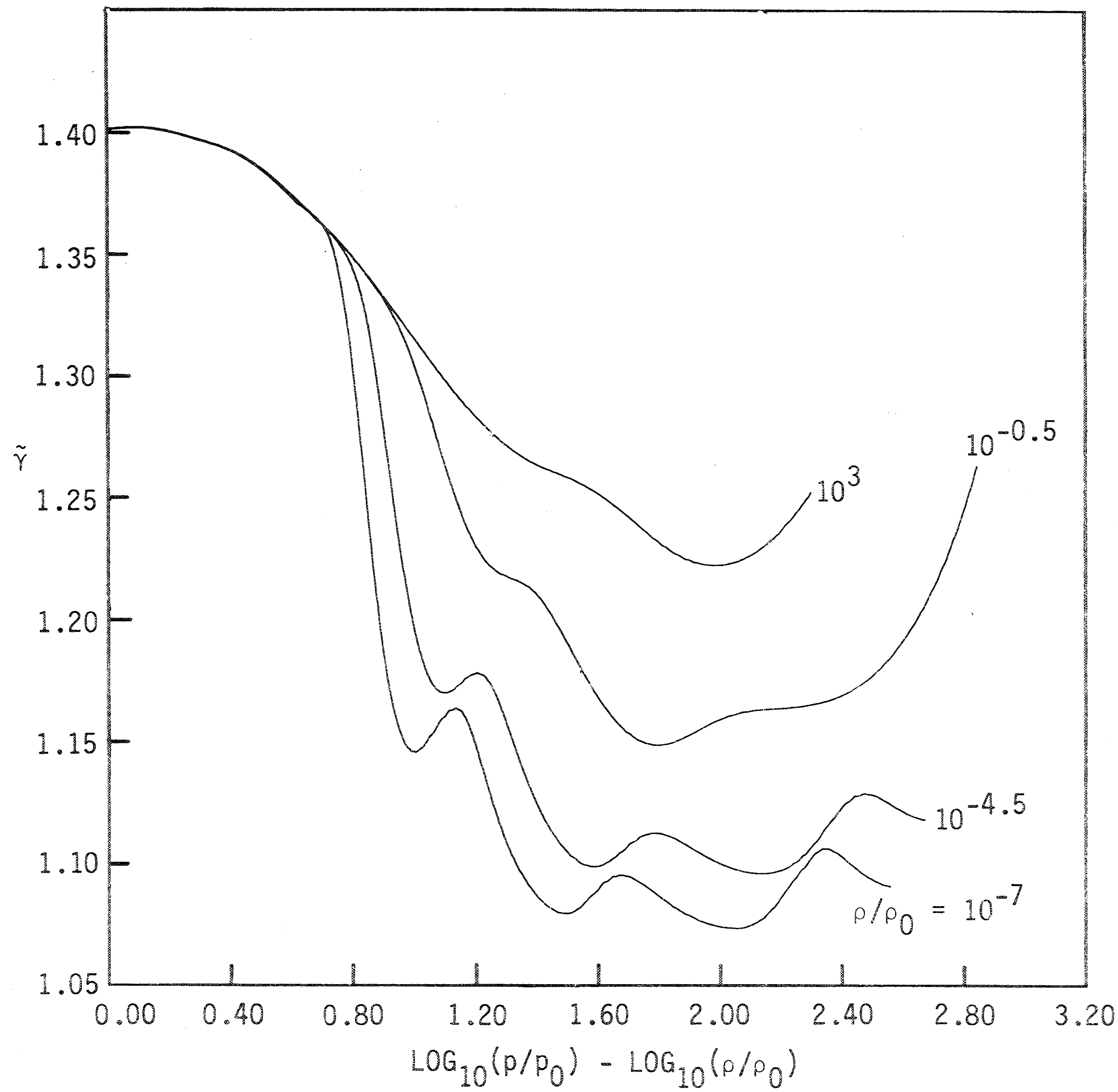

Figure 4. Variation of $\tilde{\gamma}$ with $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0)$

might be correlated with the simultaneous abrupt increases of the concentrations of ionized oxygen and nitrogen atoms and of ionized nitrogen molecules.

As a consequence of the above discussion, one is motivated to model the thermodynamic surface, in a piecewise manner, with biquadratic or bicubic polynomials joined together by exponential transition functions with or without points of inflection. This was the procedure adopted in the present study.

CONSTRUCTION OF CURVE FITS

The basic form of the variables $\tilde{\gamma}$ and $\log_{10}(T/T_0)$, plotted at constant densities as functions of $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0)$, are shown in figures 4 and 5. As mentioned previously, these curves exhibit segments of linear or quadratic functions successively connected by transition functions which are asymptotic at both ends, and may or may not include points of inflection. The fact that at least some of these transitions can be attributed to dissociation phenomena suggests the use of exponential distribution functions.

Following the method outlined by Grabau (ref. 11), one has a choice of two kernel transition functions. The first is the Fermi-Dirac function

$$\frac{1}{1 + \exp(kx)} \tag{1}$$

which represents a transition between the levels zero and unity, where the direction and rate of the transition depend on the sign and numerical magnitude of the exponential constant k. The numerator defines the upper level of the transition and may take on a variety of forms. In figure 6 the upper level of the transition is a straight line inclined to the horizontal, while the lower level is the x-axis. The transitions in figure 6 have points of inflection and, following

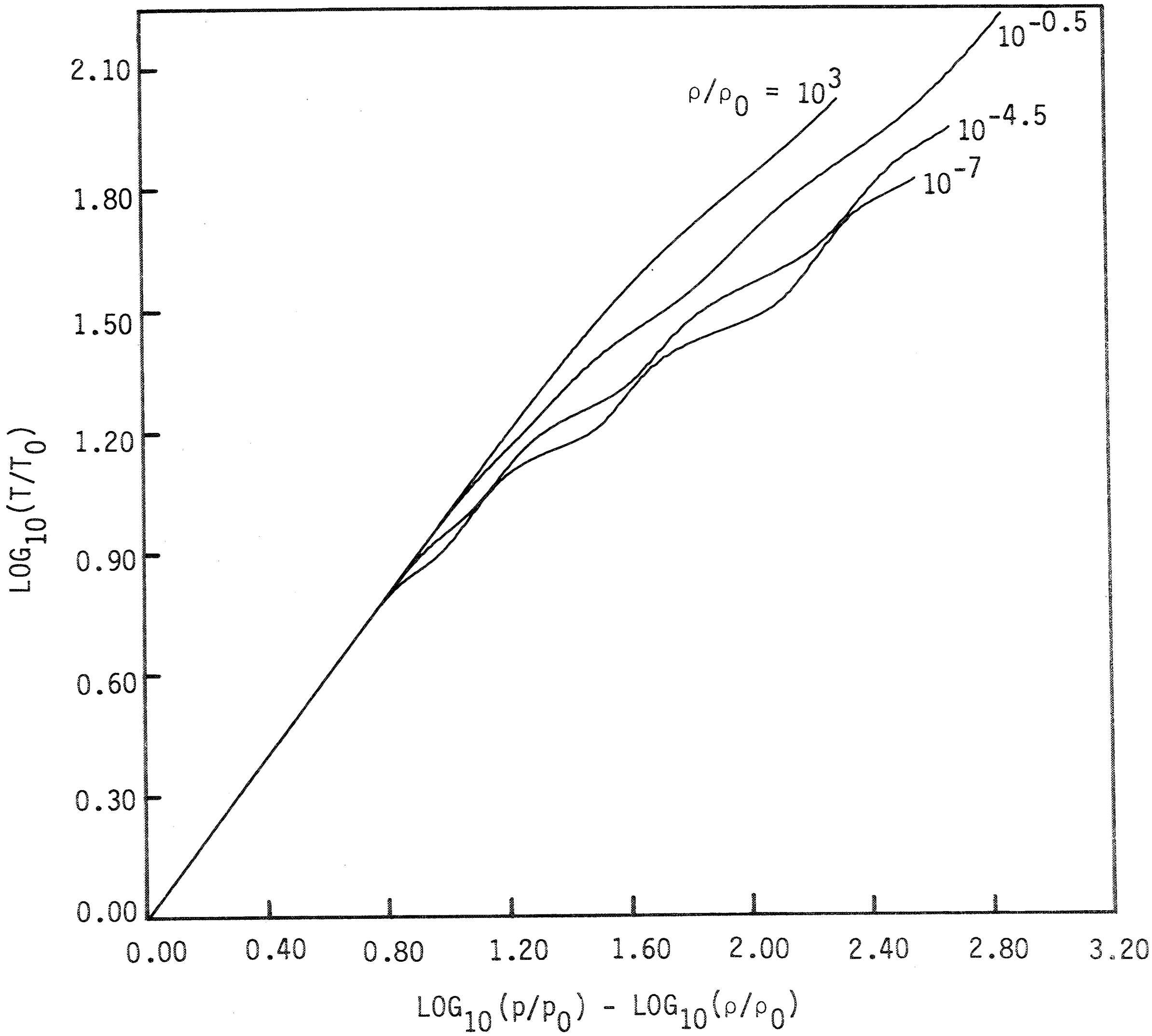

Figure 5. Variation of $\log_{10}(T/T_0)$ with $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0)$

the terminology of Grabau (ref. 11), are referred to as odd transitions.

The second type of transition function is the kernel of the Bose-Einstein distribution function

$$\frac{1}{1 - \exp(kx)} \tag{2}$$

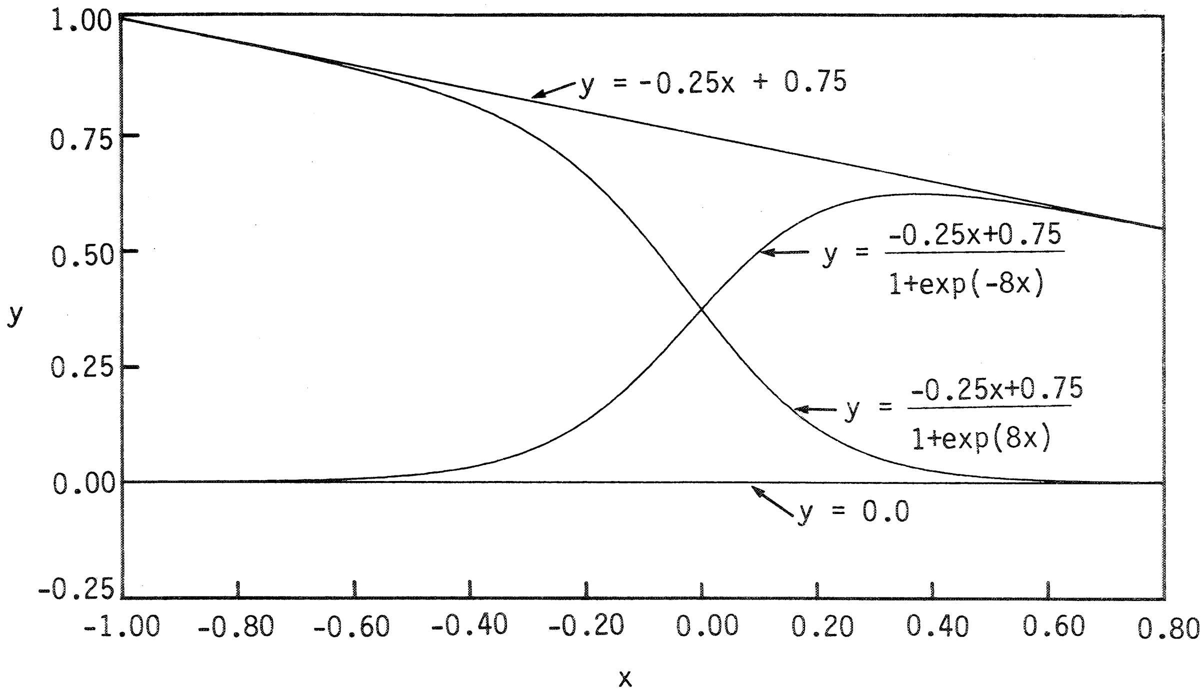

Figure 6. Two odd transition functions

which provides transitions leading from one function to another without a point of inflection and is obtained by merely changing the sign before the exponential term in the denominator of the Fermi-Dirac function. The transition function given by equation (2) is termed an even transition. Figure 7 illustrates two transitions of this kind between the x-axis and the line $y = x$, where, as before, the directions and rates of the transitions are governed by the sign and magnitude of the exponential constant k. It is important to

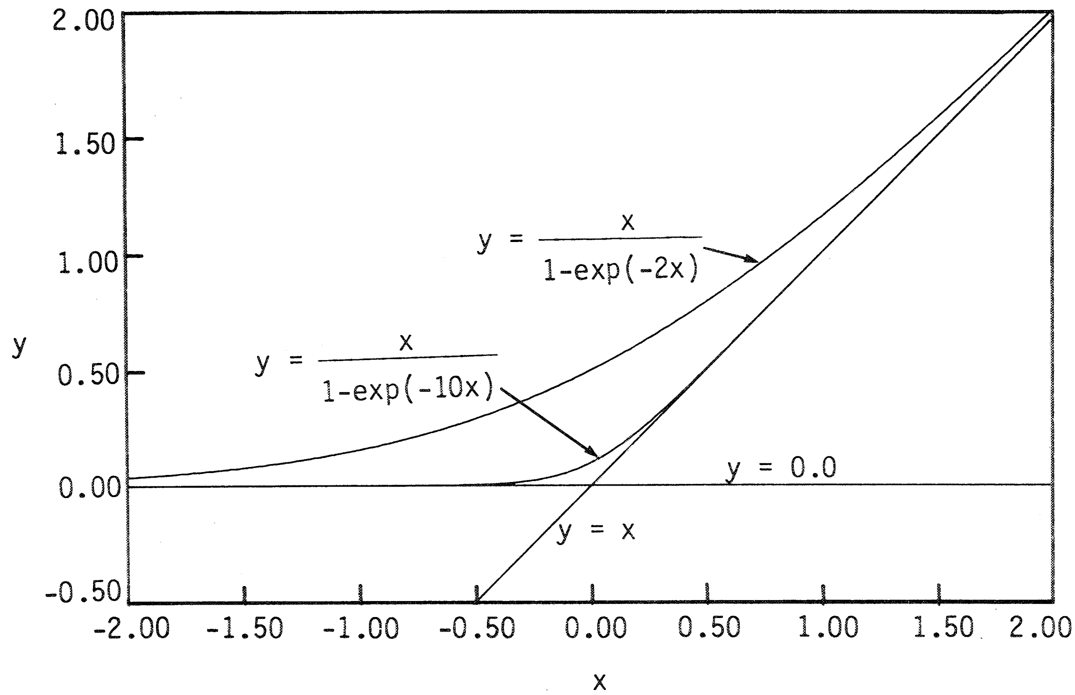

Figure 7. Two even transition functions

note that the expression for an even transition becomes an indeterminate form when x is equal to the x-coordinate of the point of intersection of the two lines bounding the transition.

In the current study, each of the thermodynamic curves was approximated by means of quadratic or incomplete cubic segments connected by odd and even transitions as described above. Almost without exception, all the curves undergo odd transitions at low densities which gradually diminish as the density increases and change to even transitions. There are two ways of applying each of these transition functions. When the path of a curve appears to move from one straight line to another, there is present an offset which can be calculated in the direction of either of the variables.

From considerations of accuracy it appears to be better to view the transition in terms of the smaller offset. Both ways of viewing the offsets involve the choice of a base line. The use of the offset in the y-direction simplifies this choice since the x-axis serves as a natural base line.

Consider the problem of determining the equation of a curve consisting of two linear segments connected by an odd transition function (fig. 8). The lower and upper line segments are given

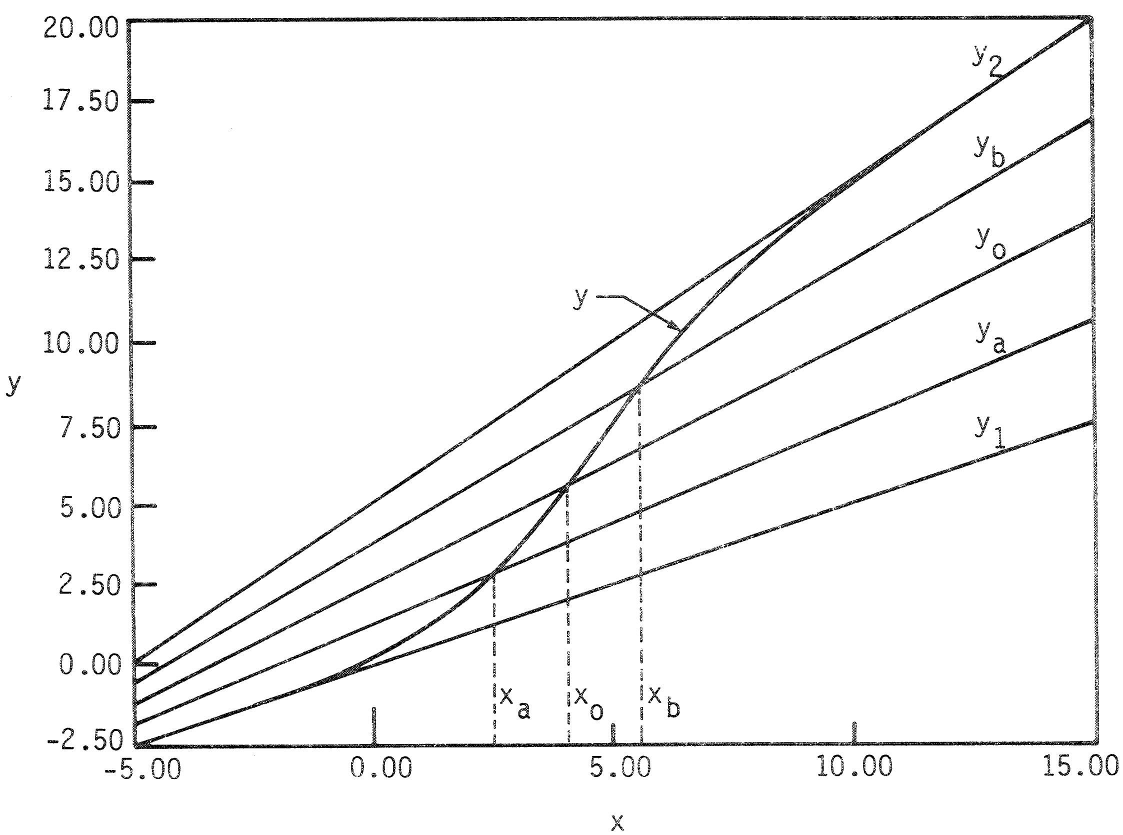

Figure 8. Construction of an odd transiton function

by

$$y_1 = a_1 x + b_1 \tag{3}$$

and

$$y_2 = a_2 x + b_2 \tag{4}$$

The y offset is their difference

$$y_2 - y_1 = (a_2 - a_1)x + (b_2 - b_1) \tag{5}$$

which becomes the numerator of the transition function. The remaining constants of the transition function can be found graphically by drawing three lines between y_1 and y_2. The median line is given by

$$y_0 = \frac{y_1 + y_2}{2} \tag{6}$$

Let y_a be the median line between y_0 and y_1 and y_b be the corresponding median line between y_0 and y_2. The center of the transition, (x_0, y_0), is the point at which the transition crosses the median line y_0. The desired transition function is then of the form

$$y = y_1 + \frac{(a_2 - a_1)x + (b_2 - b_1)}{1 + \exp[k(x - x_0)]} \tag{7}$$

The exponential constant k is found from the coordinates x_a and x_b at which the transition intersects the lines y_a and y_b. Specifically, for the intersection with the line y_a,

$$\frac{1}{1 + \exp[k(x_a - x_0)]} = 1/4 \tag{8}$$

so that

$$\exp[k(x_a - x_0)] = 3 \tag{9}$$

Solving for k yields

$$k = \frac{\log_e 3}{x_a - x_0} \tag{10}$$

From the intersection of y with y_b we get

$$k = \frac{\log_e 3}{x_0 - x_b} \tag{11}$$

This procedure obviously yields two numerical values for the constant k. However, they are found to be substantially alike in most instances.

The determination of the constants of an even transition is even simpler. In terms of the y offset, such a transition can be written in the form

$$y = \frac{a(x - x_0)}{1 - \exp[k(x - x_0)]} \tag{12}$$

where x_0 is the x coordinate of the point of intersection of the two lines bounding the transition. The value of the exponential constant k follows from the coordinate y_0 at $x = x_0$. Since the expression for y is an indeterminate form at this point, its value

is given by the ratio of the derivatives of the numerator and the denominator at this point

$$y_0 = \frac{\lim\limits_{x \to x_0} \left\{ \frac{d}{dx} [a(x - x_0)] \right\}}{\lim\limits_{x \to x_0} \left\{ \frac{d}{dx} \left\{ 1 - \exp[k(x - x_0)] \right\} \right\}} \tag{13}$$

which gives

$$k = \frac{-a}{y_0} \tag{14}$$

This approach of determining the constants of the Grabau-type transitions was extended in the present work to approximate transitions in two independent variables. The kernel of an odd transition function in three dimensions is

$$\frac{1}{1 + \exp(a_0 + a_1 x + a_2 y + a_3 xy)} \tag{15}$$

which is essentially an alternate form of

$$\frac{1}{1 + \exp[k(x - x_0)(y - y_0)]} \tag{16}$$

Equation (15) is more convenient for determining the values of the constants a_0 through a_3 as dictated by the behavior imposed on the

transition function. The general technique of determining the values of these constants differs from the approach outlined earlier and is as follows. The boundaries of the transition in the directions of the two independent variables are $x_a \leq x \leq x_b$ and $y_c \leq y \leq y_d$.

If $f_1(x,y)$ and $f_2(x,y)$ are the two surfaces limiting the transition function $f(x,y)$, then

$$f(x,y) = f_1(x,y) + \frac{f_2(x,y) - f_1(x,y)}{1 + \exp(a_0 + a_1x + a_2y + a_3xy)} \tag{17}$$

In order to ensure an accurate and smooth transition from $f_1(x,y)$ to $f_2(x,y)$ we require the quadratic expression $(a_0 + a_1x + a_2y + a_3xy)$ to behave as follows. At the lower left corner point (x_a, y_c) the quadratic expression should have a large positive value so that $f(x,y) \cong f_1(x,y)$. At the upper right corner point (x_b, y_d) the quadratic expression should have a large negative value in order to ensure that $f(x,y) \cong f_2(x,y)$. At the midpoints of the left and right boundaries, $[x_a, (y_c + y_d)/2]$ and $[x_b, (y_c + y_d)/2]$ respectively, the quadratic expression should be zero so that

$$f(x,y) \cong \frac{f_1(x,y) + f_2(x,y)}{2}.$$

These conditions yield the following four linear equations:

$$a_0 + a_1x_a + a_2y_c + a_3x_ay_c = + k \tag{18}$$

$$a_0 + a_1x_b + a_2y_d + a_3x_by_d = - k \tag{19}$$

$$a_0 + a_1x_a + a_2(y_c + y_d)/2 + a_3x_a(y_c + y_d)/2 = 0 \tag{20}$$

$$a_0 + a_1x_b + a_2(y_c + y_d)/2 + a_3x_b(y_c + y_d)/2 = 0 \tag{21}$$

where k is a positive constant (typically, $20 \leq k \leq 25$) chosen such that exp (k) and exp (-k) do not yield overflow or underflow conditions, respectively, on a computer. The constants a_0 through a_3 can now be obtained in a straightforward manner from the system of four linear equations in four unknowns (eqs. (18)-(21)).

The above method of obtaining the Grabau-type transition functions proved quite accurate in ensuring a negligible mismatch in the dependent variable over the boundaries of adjoining sub-regions. It is a merit of this stepwise method of constructing empirical equations that any part can be removed for corrections without disturbing the surface approximation as a whole.

EQUATIONS OF THE CURVE FITS

The curve fits for the various thermodynamic properties were constructed using Grabau-type transition functions, as described previously. The general form of these curve fits can be written as

$$z(x,y) = f_1(x,y) + \frac{f_2(x,y) - f_1(x,y)}{1 + \exp(k_0 + k_1 x + k_2 y + k_3 xy)} \tag{22}$$

where, in general,

$$f_1(x,y) = p_1 + p_2 x + p_3 y + p_4 xy + p_5 x^2 + p_6 y^2 + p_7 x^2 y + p_8 xy^2 + p_9 x^3 + p_{10} y^3 \tag{23}$$

and

$$f_2(x,y) - f_1(x,y) = p_{11} + p_{12}x + p_{13}y + p_{14}xy + p_{15}x^2 + p_{16}y^2 + p_{17}x^2y + p_{18}xy^2 + p_{19}x^3 + p_{20}y^3 \qquad (24)$$

The coefficients k_0 through k_3 in the denominator of the transition function in equation (22) were determined by the technique outlined in the preceding section. The coefficients p_1 through p_{20}, in equations (23) and (24), were determined by the actual curve fitting of the data from the NASA RGAS program. The exact location and number of these data points over the curve fit domain determines the accuracy of the curve fits. The points were clustered near the boundaries of the domain and the mid-region of the transition in order to ensure continuity at the boundaries and accuracy within the domain. The data from the NASA RGAS program were fitted to the equations of the curve fits by the method of least squares. A FORTRAN subroutine ULSQ was employed to perform the least square curve fitting of the data. ULSQ uses a multiple linear regression technique (ref. 17) to determine the coefficients p_1 through p_{20}.

The general form of the curve fit for each thermodynamic property is described below. As in references 10 and 15, for each of the curve fits where density is one of the independent variables, the range of ρ was subdivided into three separate regions with different coefficients being used in the curve fits for each region (fig. 9). The division lines are located at $\log_{10}(\rho/\rho_0) = -4.5$ and $\log_{10}(\rho/\rho_0) = -0.5$. In order to ensure continuity of the

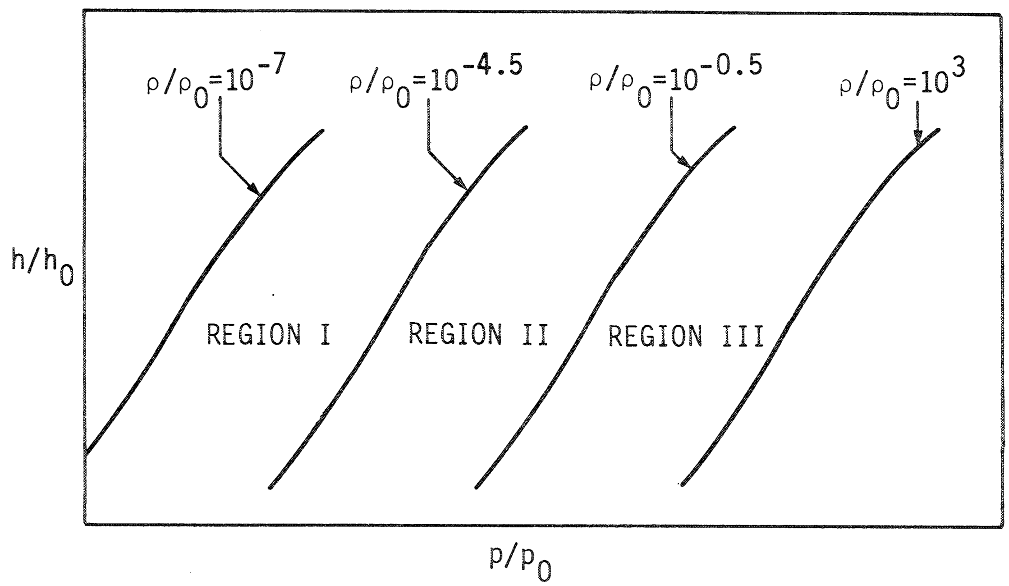

Figure 9. Division of curve fit range by density

dependent variables across these two division lines the following technique was adopted. If the choice of independent variables yielded a point within a specified band about either of these division lines, the dependent variable was linearly interpolated between the values obtained at the end points of the band.

$\underline{p = p(e,\rho)}$

For the correlation of $p = p(e,\rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted as a function of e and ρ so that p can be calculated from

$$p = \rho e(\tilde{\gamma} - 1) \tag{25}$$

The general form of the equation used for $\tilde{\gamma}$ was

$$\tilde{\gamma} = a_1 + a_2Y + a_3Z + a_4YZ + a_5Y^2 + a_6Z^2 + a_7Y^2Z + a_8YZ^2 + a_9Y^3 + a_{10}Z^3 + (a_{11} + a_{12}Y + a_{13}Z + a_{14}YZ + a_{15}Y^2 + a_{16}Z^2 + a_{17}Y^2Z + a_{18}YZ^2 + a_{19}Y^3 + a_{20}Z^3)/[1 + \exp(a_{21} + a_{22}Y + a_{23}Z + a_{24}YZ)] \tag{26}$$

where $Y = \log_{10}(\rho/\rho_0)$ and $Z = \log_{10}(e/RT_0)$. The units for ρ are kg/m^3 and the units for e are m^2/s^2. It should be noted that some of the terms appearing in the above equation are not used over the complete range of e and ρ.

<u>$a = a(e,\rho)$</u>

An exact expression for the speed of sound in terms of $\tilde{\gamma}$ was derived by Barnwell (ref. 13) and may be written as

$$a = \left[e \left\{ (\tilde{\gamma} - 1) \left[\tilde{\gamma} + \left(\frac{\partial\tilde{\gamma}}{\partial \log_e e}\right)_\rho \right] + \left(\frac{\partial\tilde{\gamma}}{\partial \log_e \rho}\right)_e \right\} \right]^{1/2} \tag{27}$$

Since complete bicubic polynomials were used for $f_1(Y,Z)$ and $f_2(Y,Z) - f_1(Y,Z)$ in equation (26) for $\tilde{\gamma}$, equation (27) was used directly for the correlation $a = a(e,\rho)$ without further corrections, unlike references 10 and 15.

$\underline{T = T(e,\rho)}$

In the calculation of $T = T(e,\rho)$, the pressure is first determined using equation (25), and then the temperature is calculated using the equation

$$\log_{10}(T/T_0) = b_1 + b_2Y + b_3Z + b_4YZ + b_5Y^2 + b_6Z^2 + b_7Y^2Z + b_8YZ^2 + b_9Y^3 + b_{10}Z^3 + (b_{11} + b_{12}Y + b_{13}Z + b_{14}YZ + b_{15}Y^2 + b_{16}Z^2 + b_{17}Y^2Z + b_{18}YZ^2 + b_{19}Y^3 + b_{20}Z^3)/[1 + \exp(b_{21} + b_{22}Y + b_{23}Z + b_{24}YZ)] \tag{28}$$

where $Y = \log_{10}(\rho/\rho_0)$, $X = \log_{10}(p/p_0)$, and $Z = X - Y$. The units for p are newtons/m^2, and the units for T are kelvin. The coefficients b_1 to b_{24} were determined in such a way as to compensate for the errors incurred in the initial calculation of pressure using equation (25).

$\underline{h = h(p,\rho)}$

For the correlation of $h = h(p,\rho)$, the ratio $\tilde{\gamma} = h/e$ was curve fitted as a function of p and ρ so that h can be calculated from

$$h = (p/\rho)\ \frac{\tilde{\gamma}}{\tilde{\gamma}-1} \tag{29}$$

The general form of the equation used for $\tilde{\gamma}$ was

$$\tilde{\gamma} = c_1 + c_2Y + c_3Z + c_4YZ + c_5Y^2 + c_6Z^2 + c_7Y^2Z + c_8YZ^2 + c_9Y^3 + c_{10}Z^3 + (c_{11} + c_{12}Y + c_{13}Z + c_{14}YZ + c_{15}Y^2 + c_{16}Z^2 + c_{17}Y^2Z + c_{18}YZ^2 + c_{19}Y^3 + c_{20}Z^3)/[1 + \exp(c_{21} + c_{22}Y + c_{23}Z + c_{24}YZ)] \tag{30}$$

where $Y = \log_{10}(\rho/\rho_0)$, $X = \log_{10}(p/p_0)$, and $Z = X - Y$. For the correlations $p = p(e,\rho)$ and $h = h(p,\rho)$, where $\tilde{\gamma}$ was the variable curve-fitted, an even transition function was used to model the transition between the perfect gas equation and the remainder of the curve fit in the lowest density region ($-7.0 \leq \log_{10}(\rho/\rho_0) \leq -4.50$). This yielded a more accurate fit than an ordinary bicubic curve without any transitions, though the latter seems intuitively more appropriate.

<u>$T = T(p,\rho)$</u>

The general form of the equation used for the correlation $T = T(p,\rho)$ was

$$\begin{aligned}\log_{10}(T/T_0) = {} & d_1 + d_2Y + d_3Z + d_4YZ + d_5Y^2 + d_6Z^2 + d_7Y^2Z + d_8YZ^2 \\ & + d_9Y^3 + d_{10}Z^3 + (d_{11} + d_{12}Y + d_{13}Z + d_{14}YZ + d_{15}Y^2 \\ & + d_{16}Z^2 + d_{17}Y^2Z + d_{18}YZ^2 + d_{19}Y^3 + d_{20}Z^3)/[1 + \exp(d_{21} \\ & + d_{22}Y + d_{23}Z + d_{24}YZ)]\end{aligned} \tag{31}$$

where $Y = \log_{10}(\rho/\rho_0)$, $X = \log_{10}(p/p_0)$ and $Z = X - Y$

<u>$s = s(e,\rho)$</u>

For the correlation of $s = s(e,\rho)$, the general form of the equation used was

$$\begin{aligned}\frac{s}{R} = {} & e_1 + e_2Y + e_3Z + e_4YZ + e_5Y^2 + e_6Z^2 + e_7Y^2Z + e_8YZ^2 \\ & + e_9Y^3 + e_{10}Z^3\end{aligned} \tag{32}$$

where $Y = \log_{10}(\rho/\rho_0)$ and $Z = \log_{10}(e/RT_0)$. The units for s are $m^2/s^2 - K$. As is evident from equation (32), Grabau transition functions were not necessary for this curve fit.

$\underline{\rho = \rho(p,s)}$

Unlike the preceding curve fits in which density is one of the independent variables, the domain of the curve fit $\rho = \rho(p,s)$, as well as the curve fits $e = e(p,s)$, and $a = a(p,s)$, cannot be divided into subdomains on the basis of density. For reasons of accuracy, it was necessary to subdivide the domain in terms of s as seen in figure 10.

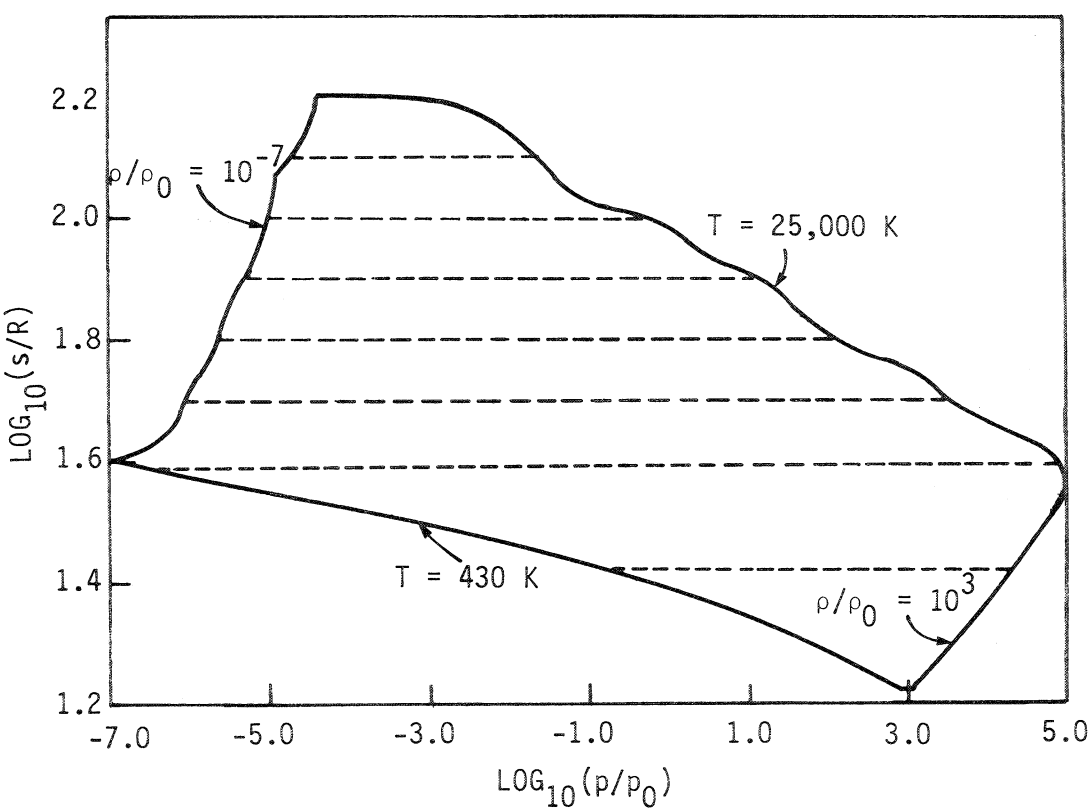

Figure 10. Division of curve fit range by entropy

The general form of the equation used for the correlation $\rho = \rho(p,s)$ was

$$\log_{10}(\rho/\rho_0) = f_1 + f_2Y + f_3Z + f_4YZ + f_5Y^2 + f_6Z^2 + f_7Y^2Z + f_8YZ^2 + f_9Y^3 + f_{10}Z^3 + (f_{11} + f_{12}Y + f_{13}Z + f_{14}YZ + f_{15}Y^2 + f_{16}Z^2 + f_{17}Y^2Z + f_{18}YZ^2 + f_{19}Y^3 + f_{20}Z^3)/[1 + \exp(f_{21} + f_{22}Y + f_{23}Z)] \quad (33)$$

where $Y = \log_{10}(s/R)$, $X = \log_{10}(p/p_0)$, and $Z = X - Y$. The units for s are $m^2/sec^2 - K$. The denominator of the Grabau transition function in equation (33) is linear in Y and Z and not quadratic.

$e = e(p,s)$

For the correlation of $e = e(p,s)$, the general form of the curve-fit equation was

$$\log_{10}(e/RT_0) = g_1 + g_2Y + g_3Z + g_4YZ + g_5Y^2 + g_6Z^2 + g_7Y^2Z + g_8YZ^2 + g_9Y^3 + g_{10}Z^3 + (g_{11} + g_{12}Y + g_{13}Z + g_{14}YZ + g_{15}Y^2 + g_{16}Z^2 + g_{17}Y^2Z + g_{18}YZ^2 + g_{19}Y^3 + g_{20}Z^3)/[1 + \exp(g_{21} + g_{22}Y + g_{23}Z)] \quad (34)$$

where $Y = \log_{10}(s/R)$, $X = \log_{10}(p/p_0)$, and $Z = X - Y$.

$\underline{a = a(p,s)}$

For the correlation of $a = a(p,s)$, the general form of the equation used was

$$\log_{10}(a/a_0) = h_1 + h_2Y + h_3Z + h_4YZ + h_5Y^2 + h_6Z^2 + h_7Y^2Z + h_8YZ^2 + h_9Y^3 + h_{10}Z^3 + (h_{11} + h_{12}Y + h_{13}Z + h_{14}YZ + h_{15}Y^2 + h_{16}Z^2 + h_{17}Y^2Z + h_{18}YZ^2 + h_{19}Y^3 + h_{20}Z^3)/[1 + \exp(h_{21} + h_{22}Y + h_{23}Z)] \tag{35}$$

where $Y = \log_{10}(s/R)$, $X = \log_{10}(p/p_0)$, and $Z = X - Y$. The units of a are m/sec.

RESULTS AND CONCLUSIONS

New simplified curve fits for the thermodynamic properties of equilibrium air were constructed using the procedures described in the preceding chapters. Comparisons of the curve fits $p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(e,\rho)$, $s = s(e,\rho)$, $T = T(p,\rho)$, $h = h(p,\rho)$, $\rho = \rho(p,s)$, $e = e(p,s)$ and $a = a(p,s)$ with the original NASA RGAS program are shown in figures 11 to 19. The following procedure was employed in making the comparisons for the first four curve fits. First, p and ρ data were supplied as input to the NASA RGAS program and e was computed. Then, this e and the original ρ were inputed into the TGAS1 subroutine to obtain p, a and T and into the TGAS2 subroutine to obtain s. As a result of this procedure, $\log_{10}(p/p_0)$

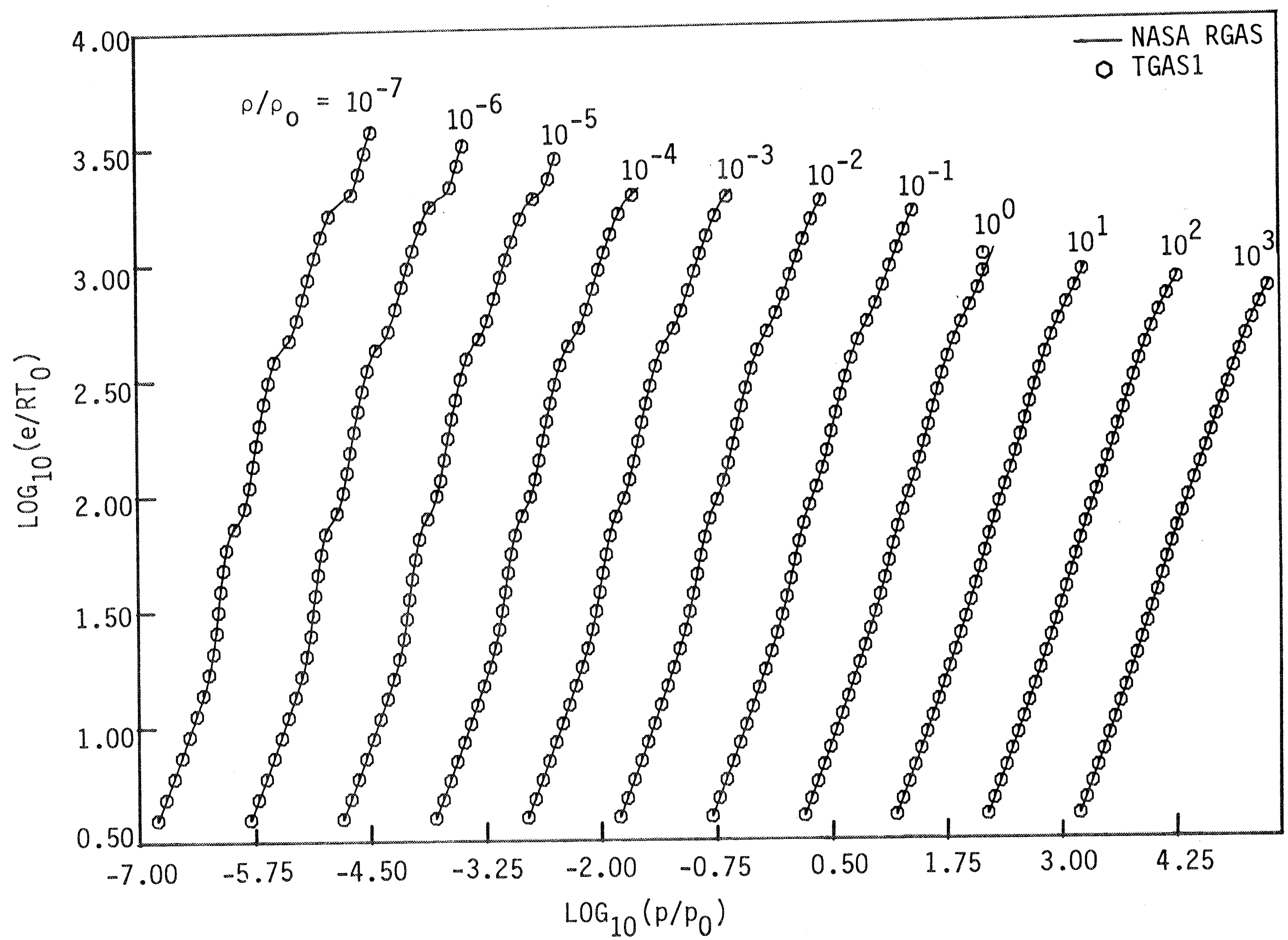

Figure 11. Comparison of curve fits for $p = p(e,\rho)$

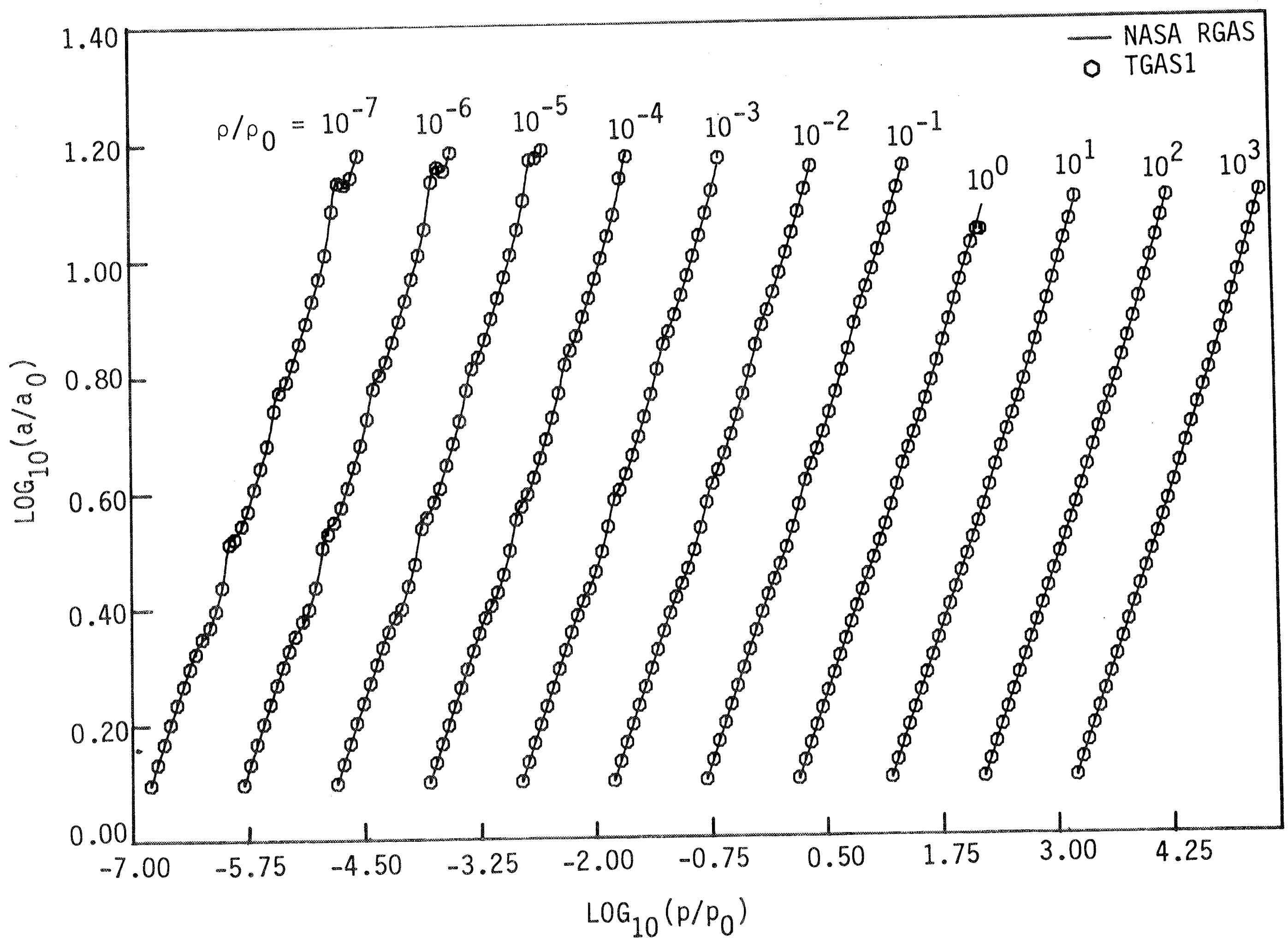

Figure 12. Comparison of curve fits for $a = a(e,\rho)$

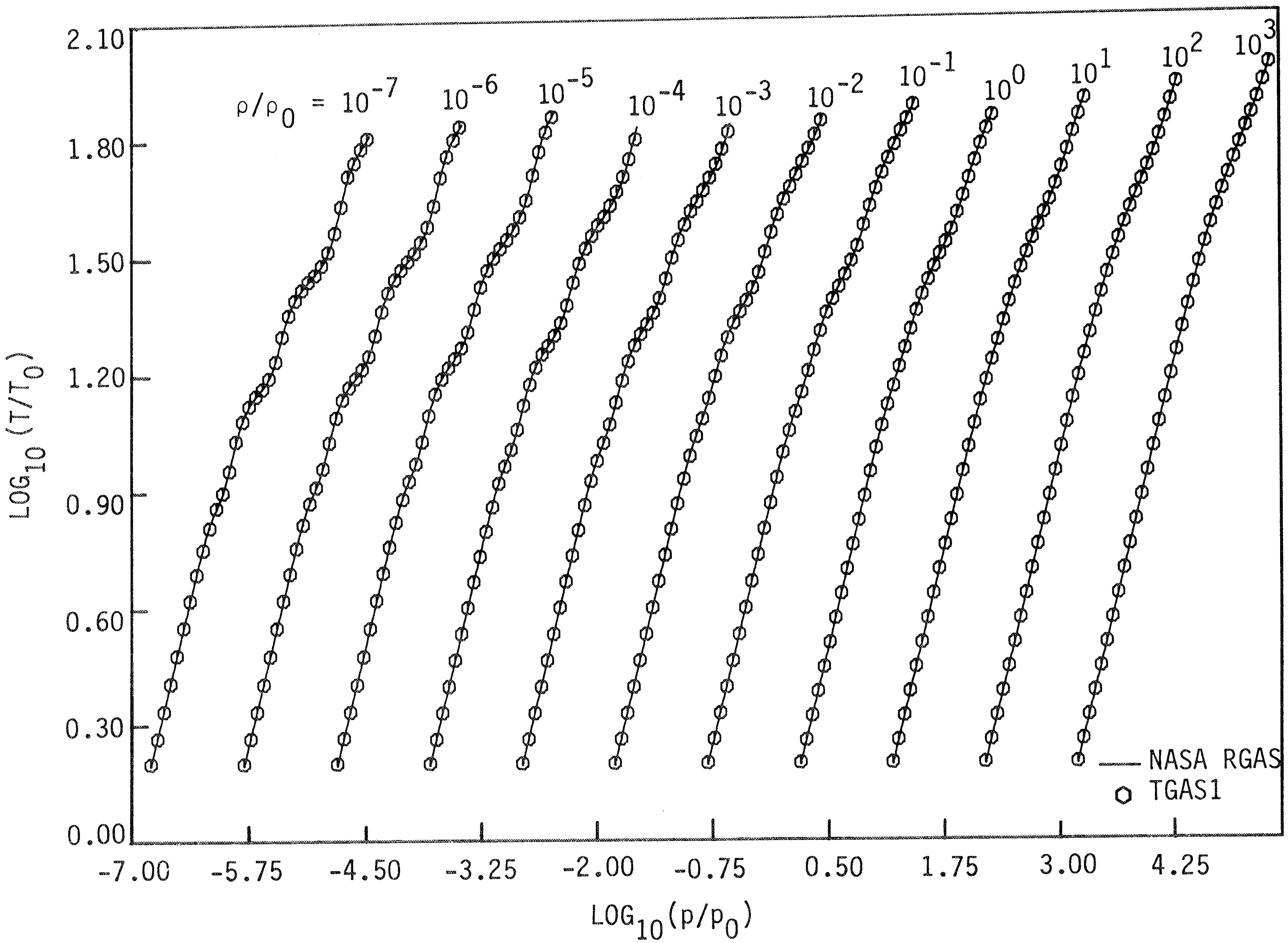

Figure 13. Comparison of curve fits for $T = T(e,\rho)$

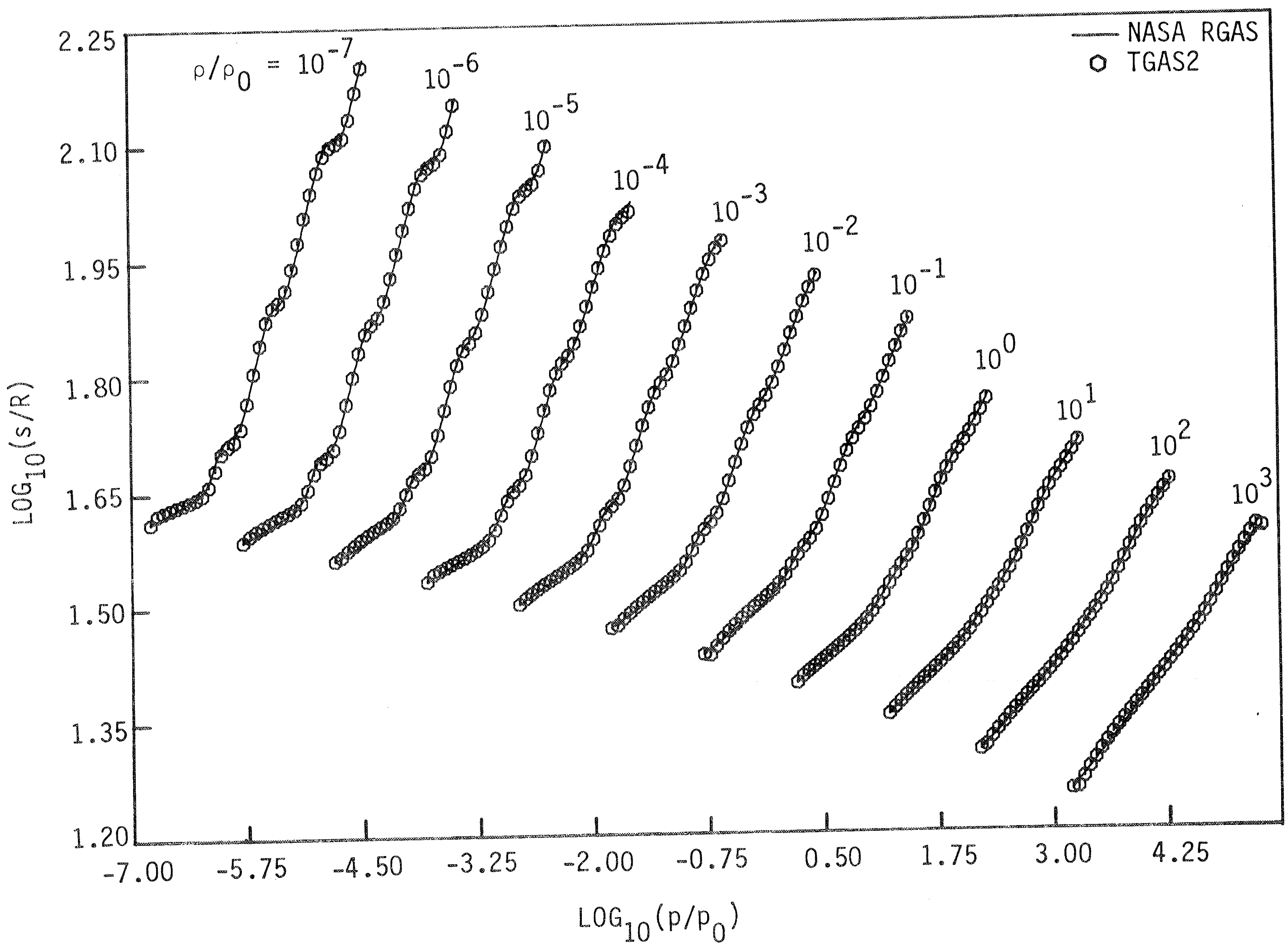

Figure 14. Comparison of curve fits for $s = s(e,\rho)$

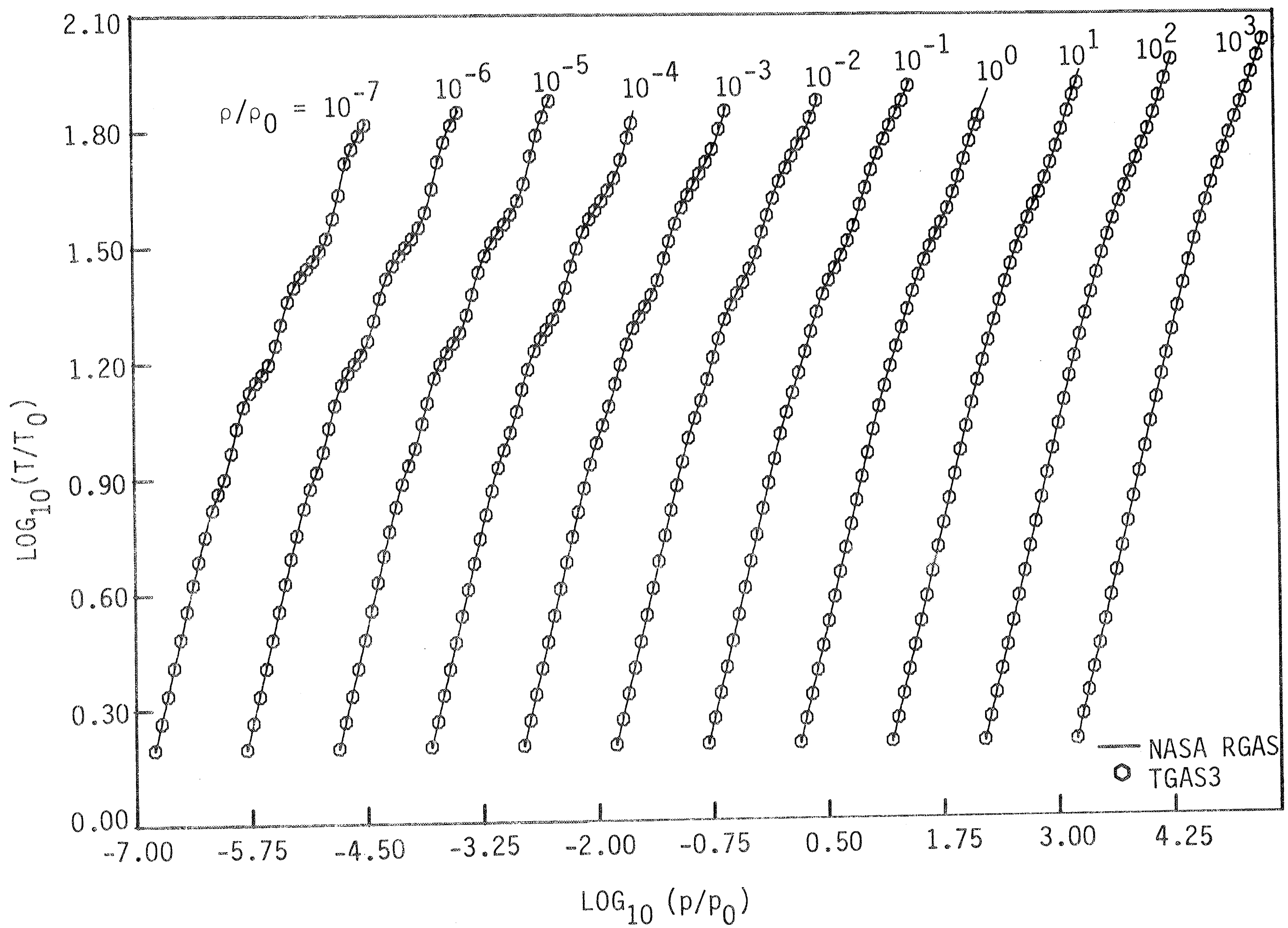

Figure 15. Comparison of curve fits for $T = T(p,\rho)$

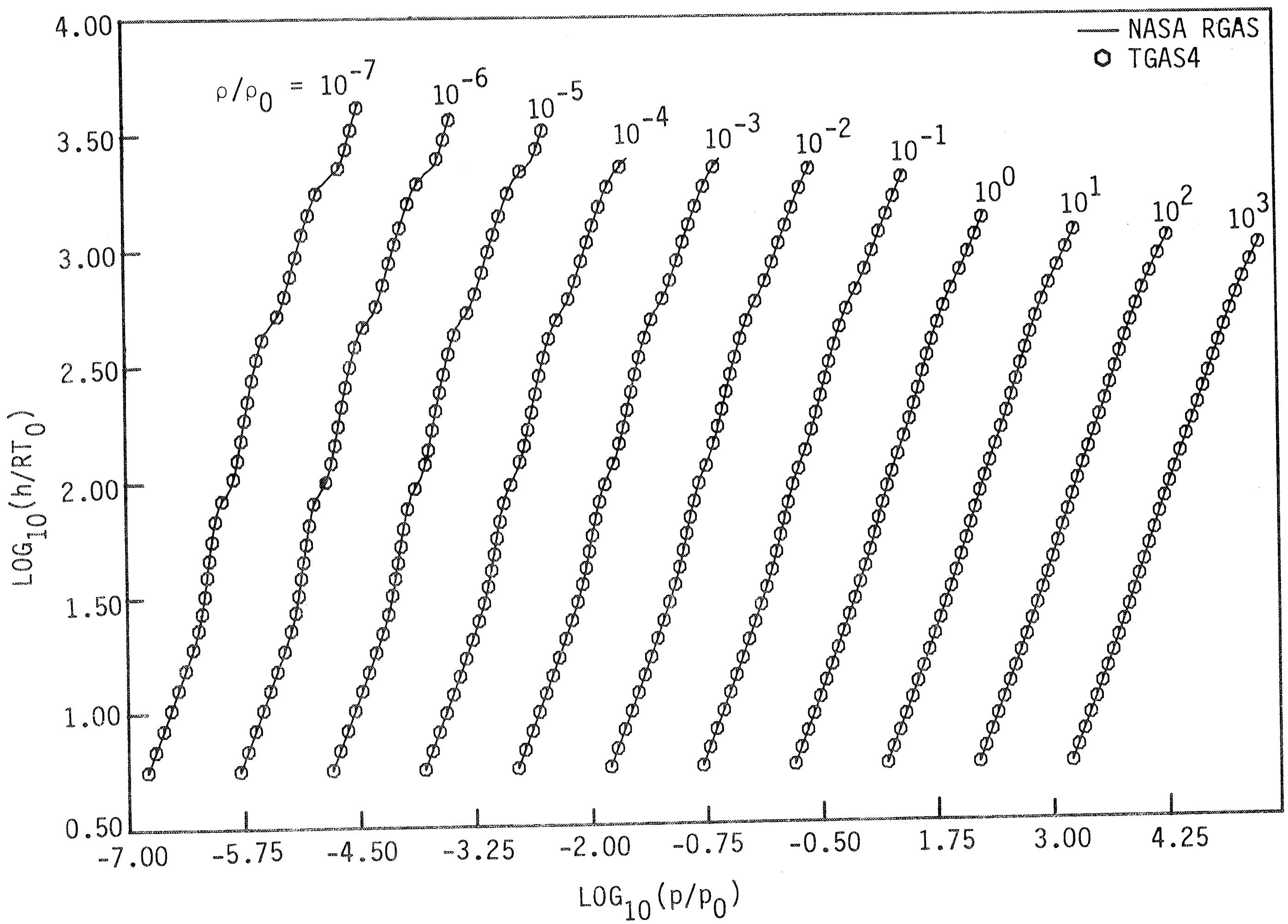

Figure 16. Comparison of curve fits for $h = h(p,\rho)$

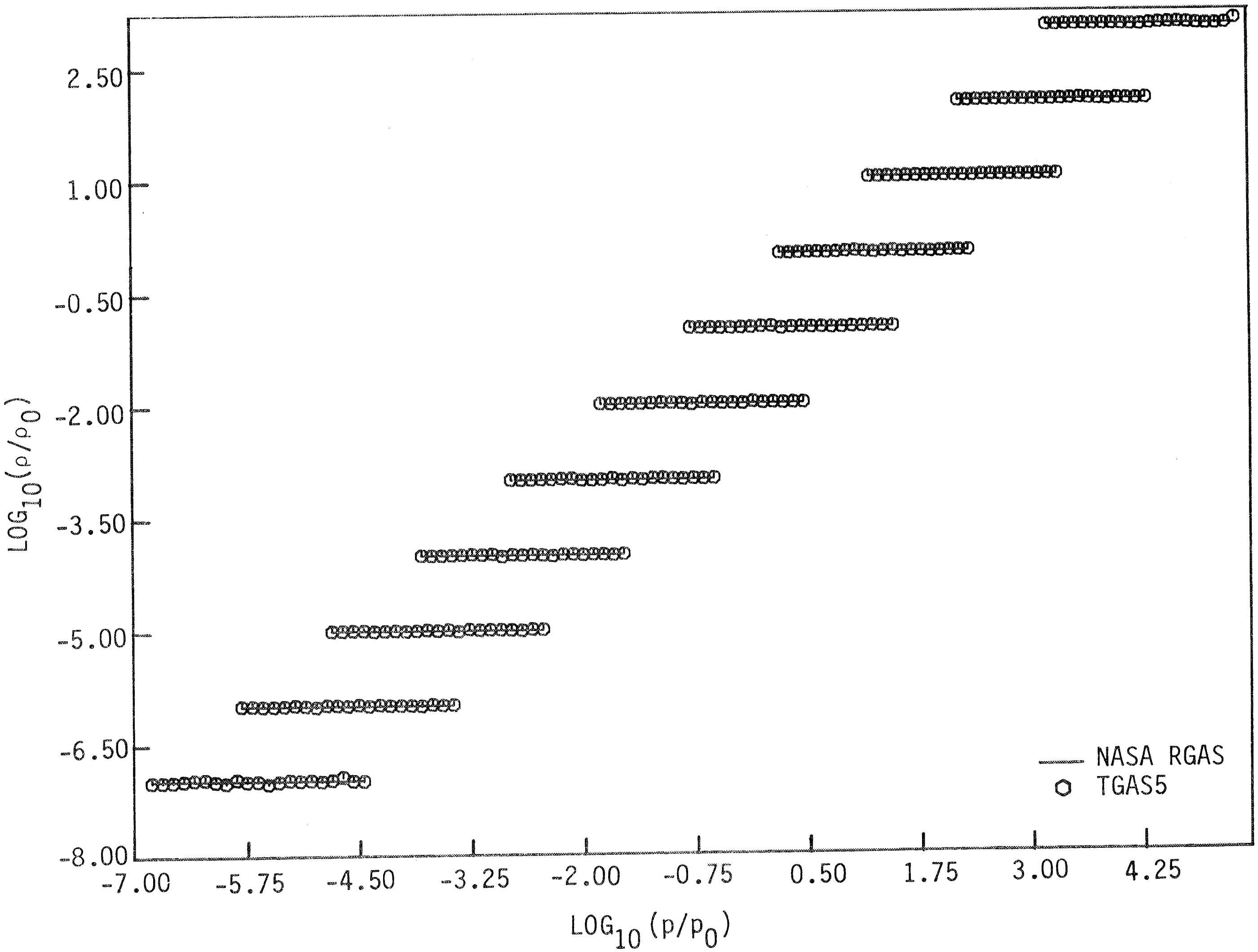

Figure 17. Comparison of curve fits for $\rho = \rho(p,s)$

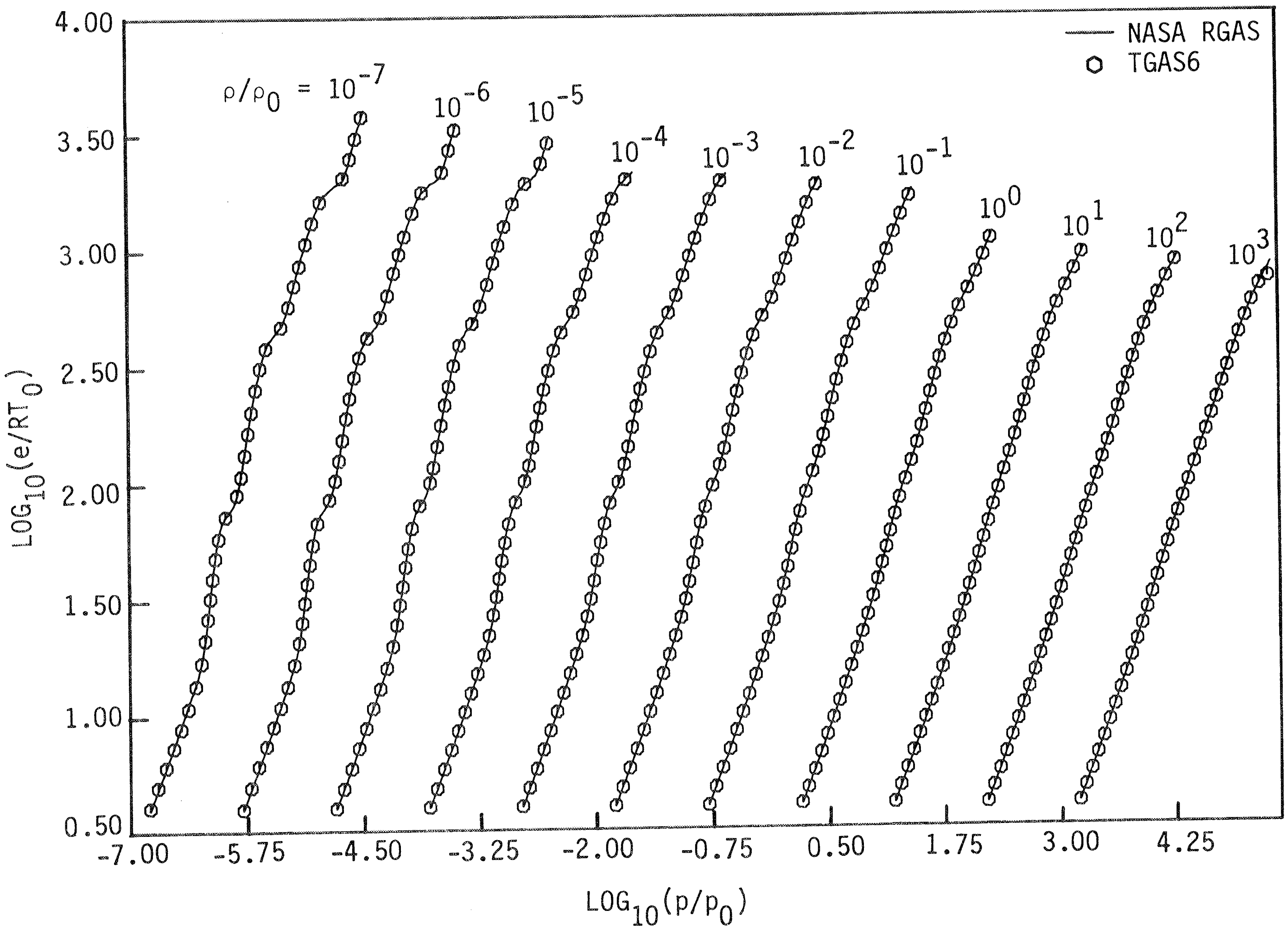

Figure 18. Comparison of curve fits for e = e(p,s)

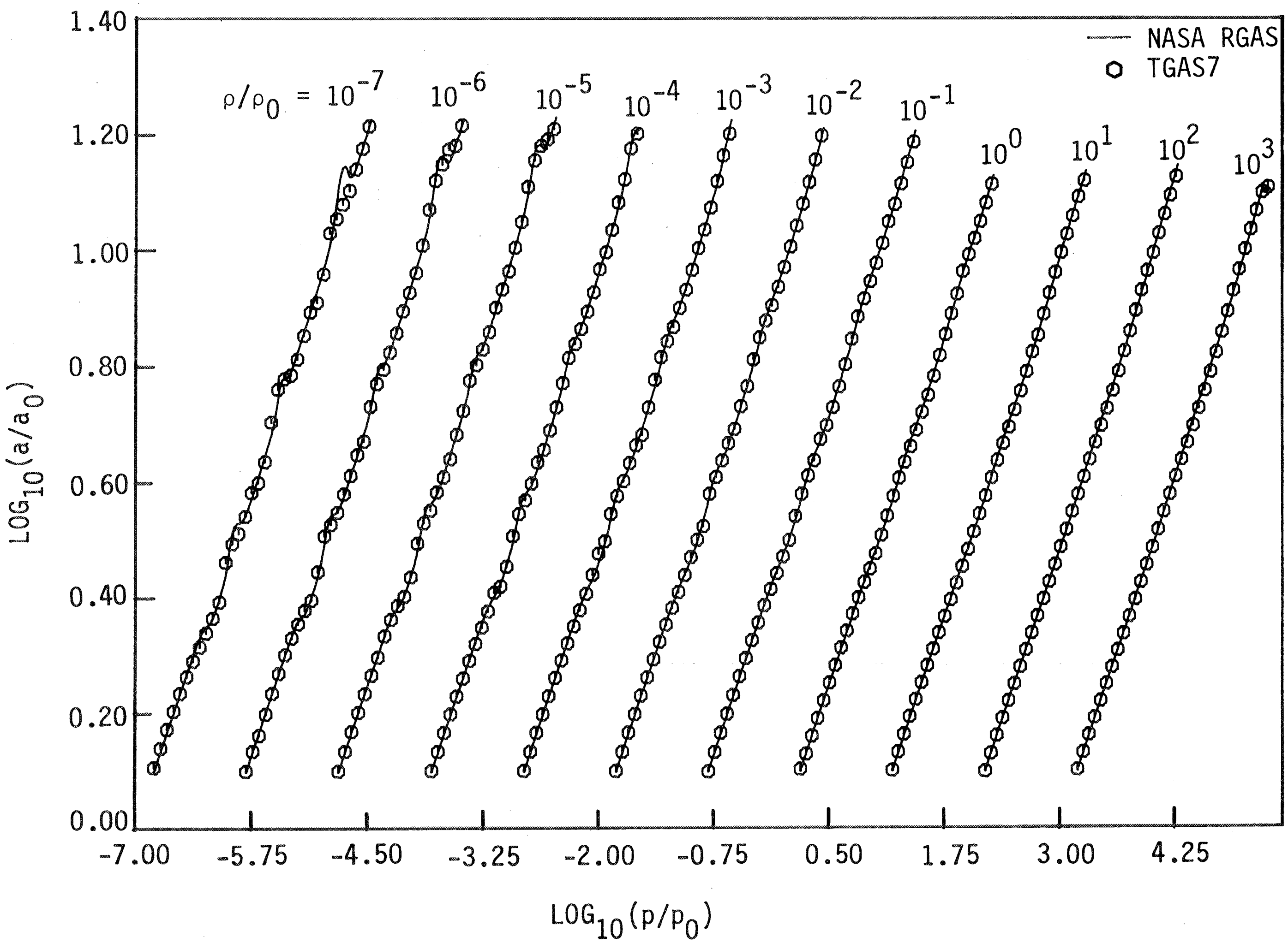

Figure 19. Comparison of curve fits for a = a(p,s)

is plotted as one of the independent variables in figures 11 to 14. The same p and ρ data used above was also employed in the comparisons for the curve fits $T = T(p,\rho)$ and $h = h(p,\rho)$.

The method adopted for the comparisons of $\rho = \rho(p,s)$, $e = e(p,s)$ and $a = a(p,s)$ with the NASA RGAS program were quite similar to that for the first four curve fits. First, p and ρ were supplied to the NASA RGAS program which yielded s. This s and the original p were inputed into TGAS5 to obtain ρ, into TGAS6 to obtain e and into TGAS7 to obtain a, respectively.

The above comparisons have been presented graphically to provide a qualitative overview of the accuracy of the curve fits. However, as figures 11 to 19 indicate, these graphical comparisons were restricted to points lying on 11 constant density lines ranging from 10^{-7} to 10^3 amagats. In order to ensure the validity and accuracy of the curve fits across the entire domain, a more comprehensive accuracy test was carried out. The new curve fits were compared with the NASA RGAS program for relative accuracies at approximately 22,000 data points. These test points were chosen to span the entire density range from 10^{-7} to 10^3 amagats and temperatures varying from 273 K to 25,000 K. The results of these comprehensive accuracy checks are presented in tables 1 to 9. For the curve fits $p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(e,\rho)$, $T = T(p,\rho)$ and $h = h(p,\rho)$, comparisons with the curve fits of reference 15 are also presented in the tables. The numbers in the first line of the tables are percentages of compared values with errors greater than the given value. The accuracies of the present curve fits are substantially improved over the accuracies of the previous curve fits appearing in reference 15. The

TABLE 1. ACCURACY OF TGAS1 FOR $p = p(e,\rho)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS1	28.43	10.63	1.01	0.03	0	0	0	0	0	0	0
TGAS	68.29	42.87	17.51	6.69	1.49	0.24	0.14	0.04	0.01	0	0

Total Number of Data Points = 22,239

Maximum Error in TGAS1 = 3.93%

at $\log_{10}(\rho/\rho_0) = -4.0$; $\log_{10}(e/RT_0) = 3.28$

and $T = 1.47 \times 10^4$ K

Maximum Error in TGAS = 9.0%

at $\log_{10}(\rho/\rho_0) = -4.5$; $\log_{10}(e/RT_0) = 2.236$

and $T = 4.53 \times 10^3$ K

TABLE 2. ACCURACY OF TGAS1 FOR $a = a(e,\rho)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS1	20.94	5.75	0.70	0.09	0.02	0	0	0	0	0	0
TGAS	60.67	27.21	5.17	0.98	0.11	0	0	0	0	0	0

Total Number of Data Points = 22,239

Maximum Error in TGAS1 = 4.48%

at $\log_{10}(\rho/\rho_0) = -3.0$; $\log_{10}(e/RT_0) = 3.318$

and $T = 2.0 \times 10^4$ K

Maximum Error in TGAS = 4.94%

at $\log_{10}(\rho/\rho_0) = -7.0$; $\log_{10}(e/RT_0) = 3.279$

and $T = 1.25 \times 10^4$ K

TABLE 3. ACCURACY OF TGAS1 FOR $T = T(e,\rho)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS1	34.11	10.87	0.58	0.10	0.01	0	0	0	0	0	0
TGAS	63.82	34.74	9.51	2.43	0.59	0.19	0.09	0.04	0.02	0	0

Total Number of Data Points = 22,239

Maximum Error in TGAS1 = 4.36%

at $\log_{10}(\rho/\rho_0) = -4.0$; $\log_{10}(e/RT_0) = 3.28$

and $T = 1.47 \times 10^4$ K

Maximum Error in TGAS = 8.8%

at $\log_{10}(\rho/\rho_0) = -0.625$; $\log_{10}(e/RT_0) = 3.255$

and $T = 2.4 \times 10^4$ K

TABLE 4. ACCURACY OF TGAS2 FOR $s = s(e,\rho)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	$\geq$10%
TGAS2	49.77	15.95	0.56	0	0	0	0	0	0	0	0

Total Number of Data Points = 21,975

Maximum Error in TGAS2 = 2.51%

at $\log_{10}(\rho/\rho_0) = -0.625$; $\log_{10}(e/RT_0) = 0.657$

and $T = 4.89 \times 10^2$ K

TABLE 5. ACCURACY OF TGAS3 FOR $T = T(p, \rho)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS3	22.89	8.24	0.22	0.03	0	0	0	0	0	0	0
TGAS	58.82	28.75	4.89	0.96	0.16	0.04	0	0	0	0	0

Total Number of Data Points = 22,239

Maximum Error in TGAS3 = 3.9%

at $\log_{10}(\rho/\rho_0) = -3.25$; $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0) = 2.58$

and $T = 2.4 \times 10^4$ K

Maximum Error in TGAS = 5.71%

at $\log_{10}(\rho/\rho_0) = -0.625$; $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0) = 2.44$

and $T = 2.3 \times 10^4$ K

TABLE 6. ACCURACY OF TGAS4 FOR $h = h(p,\rho)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS4	23.85	7.65	0.55	0.04	0	0	0	0	0	0	0
TGAS	67.45	40.36	13.65	4.78	1.56	0.46	0.16	0	0	0	0

Total Number of Data Points = 22,239

Maximum Error in TGAS4 = 3.44%

at $\log_{10}(\rho/\rho_0) = -7.0$; $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0) = 2.60$

and $T = 1.91 \times 10^4$ K

Maximum Error in TGAS = 6.56%

at $\log_{10}(\rho/\rho_0) = -4.5$; $\log_{10}(p/p_0) - \log_{10}(\rho/\rho_0) = 1.01$

and $T = 2.47 \times 10^3$ K

TABLE 7. ACCURACY OF TGAS5 FOR $\rho = \rho(p,s)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS5	62.06	40.25	14.97	4.46	0.98	0.35	0.03	0.01	0	0	0

Total Number of Data Points = 21,030

Maximum Error in TGAS5 = 7.58%

at $\log_{10}(\rho/\rho_0) = -6.625$; $\log_{10}(e/RT_0) = 3.30$

and $T = 1.42 \times 10^4$ K

TABLE 8. ACCURACY OF TGAS6 FOR $e = e(p,s)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS6	39.52	22.68	5.45	0.04	0.01	0	0	0	0	0	0

Total Number of Data Points = 21,030

Maximum Error in TGAS6 = 4.5%

at $\log_{10}(\rho/\rho_0) = 2.875$; $\log_{10}(e/RT_0) = 2.85$

and $T = 2.46 \times 10^4$ K

TABLE 9. ACCURACY OF TGAS7 FOR $a = a(p,s)$

ERROR	0.5%	1%	2%	3%	4%	5%	6%	7%	8%	9%	≥10%
TGAS6	50.98	26.28	5.74	1.67	0.48	0.08	0	0	0	0	0

Total Number of Data Points = 21,030

Maximum Error in TGAS7 = 6.1%

at $\log_{10}(\rho/\rho_0) = -2.375$; $\log_{10}(e/RT_0) = 2.39$

and $T = 6.05 \times 10^3$ K

somewhat higher percentage errors in the curve fits with p and s as independent variables can be attributed to the fact that a line of constant s spans the entire density range, sometimes necessitating the use of two Grabau-type transition functions. Requiring a minimal mismatch across the junctions of these transition functions resulted in a relative loss of accuracy. However, these latter curve fits are well within the accuracy limits required for most engineering applications.

One of the primary objectives of this research effort was to minimize the discontinuities in the dependent variables across juncture points of the curve fits (fig. 20). Comparisons of the dependent variables at juncture points of the curve fits for $p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(e,\rho)$, $T = T(p,\rho)$ and $h = h(p,\rho)$ are presented in tables 10 to 14. These new curve fits show a substantial improvement in continuity at the juncture points when compared with the previous curve fits. For the curve fits where p and s are the independent variables, it was very difficult to maintain continuity at the juncture points. This was due to the manner in which the domain was subdivided to obtain the piecewise approximating functions. However, discontinuities were kept to a minimum with average mismatches of 2.4% for $\rho = (p,s)$, 1.2% for $a = a(p,s)$ and 2.0% for $e = e(p,s)$.

A comparison of the relative computer times required for the new curve fit subroutines and the NASA RGAS program on the NAS 9160 computer are given in table 15. The new TGAS1 subroutine for determining $p = p(e,\rho)$, $a = a(e,\rho)$ and $T = T(e,\rho)$ is 2.4 times faster than the NASA RGAS subroutine. The previous TGAS subroutine

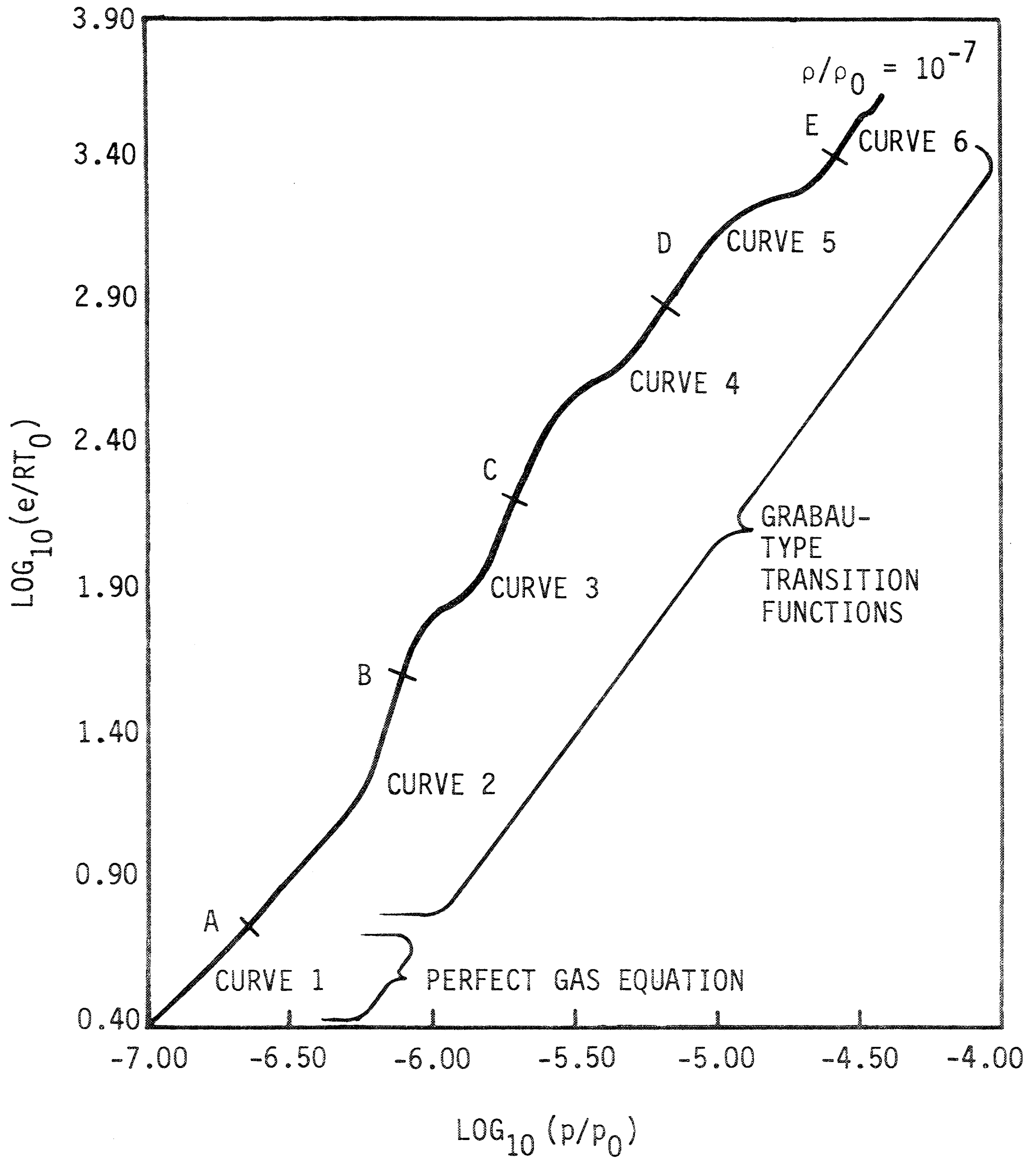

Figure 20. Example curve fit for $p = p(e,\rho)$

(ref. 15) for the same curve fits is 3.4 times faster than the NASA RGAS subroutine. The new TGAS3 subroutine for $T = T(p,\rho)$ is 2.7 times faster than the NASA RGAS subroutine as compared to the

TABLE 10. COMPARISON OF VARIABLES AT JUNCTURE POINTS FOR THE CURVE FIT $p = p(e,\rho)$

Density ratio ρ/ρ_0	POINT A		POINT B		POINT C		POINT D		POINT E	
	lower	upper	lower	upper	lower	upper	lower	upper	lower	upper
10^{-7}	1.79×10^{-2}	1.81×10^{-2}	7.32×10^{-2}	7.38×10^{-2}	1.90×10^{-1}	1.90×10^{-1}	8.72×10^{-1}	8.72×10^{-1}	2.62×10^{0}	2.63×10^{0}
10^{-6}	1.79×10^{-1}	1.80×10^{-1}	7.81×10^{-1}	7.78×10^{-1}	2.01×10^{0}	2.03×10^{0}	9.53×10^{0}	9.63×10^{0}	2.86×10^{1}	2.89×10^{1}
10^{-5}	1.79×10^{0}	1.80×10^{0}	8.17×10^{0}	8.19×10^{0}	2.16×10^{1}	2.18×10^{1}	1.05×10^{2}	1.06×10^{2}	3.15×10^{2}	3.16×10^{2}
10^{-4}	1.80×10^{1}	1.81×10^{1}	8.67×10^{1}	8.70×10^{1}	2.40×10^{2}	2.43×10^{2}	9.78×10^{2}	9.79×10^{2}	1.80×10^{2}	1.81×10^{2}
10^{-3}	1.80×10^{2}	1.81×10^{2}	9.12×10^{2}	9.13×10^{2}	2.61×10^{3}	2.63×10^{3}	1.09×10^{4}	1.09×10^{4}		
10^{-2}	1.80×10^{3}	1.81×10^{3}	9.51×10^{3}	9.51×10^{3}	2.83×10^{4}	2.84×10^{4}	1.23×10^{5}	1.23×10^{5}		
10^{-1}	1.80×10^{4}	1.81×10^{4}	9.80×10^{4}	9.81×10^{4}	3.08×10^{5}	3.08×10^{5}	1.39×10^{6}	1.39×10^{6}		
10^{0}	1.80×10^{5}	1.81×10^{5}	1.36×10^{6}	1.36×10^{6}	4.11×10^{6}	4.11×10^{6}				
10^{1}	1.80×10^{6}	1.81×10^{6}	1.41×10^{7}	1.41×10^{7}	4.50×10^{7}	4.54×10^{7}				
10^{2}	1.80×10^{7}	1.81×10^{7}	1.45×10^{8}	1.43×10^{8}	4.91×10^{8}	5.00×10^{8}				
10^{3}	1.80×10^{8}	1.83×10^{8}	1.48×10^{9}	1.45×10^{9}	5.42×10^{9}	5.53×10^{9}				

TABLE 11. COMPARISON OF VARIABLES AT JUNCTURE POINTS FOR THE CURVE FIT $a = a(e, \rho)$

Density ratio ρ/ρ_0	POINT A		POINT B		POINT C		POINT D		POINT E	
	lower	upper	lower	upper	lower	upper	lower	upper	lower	upper
10^{-7}	440	441	769	790	1250	1260	2718	2733	4731	4715
10^{-6}	440	439	808	814	1291	1307	2857	2871	4983	5016
10^{-5}	440	438	831	841	1343	1359	3021	3029	5259	5287
10^{-4}	441	440	869	874	1429	1441	2923	2925		
10^{-3}	441	440	902	904	1498	1506	3115	3116		
10^{-2}	441	440	932	932	1573	1578	3337	3341		
10^{-1}	441	441	957	957	1655	1656	3596	3602		
10^{0}	442	441	1120	1118	1924	1924				
10^{1}	442	440	1149	1145	2027	2039				
10^{2}	442	440	1171	1164	2141	2166				
10^{3}	442	441	1188	1179	2287	2312				

TABLE 12. COMPARISON OF VARIABLES AT JUNCTURE POINTS FOR THE CURVE FIT $T = T(e,\rho)$

Density ratio ρ/ρ_o	POINT A		POINT B		POINT C		POINT D		POINT E	
	lower	upper	lower	upper	lower	upper	lower	upper	lower	upper
10^{-7}	486	481	2112	2091	4033	4034	7868	7869		
10^{-6}	486	482	2181	2168	4283	4284	8471	8479		
10^{-5}	486	484	2243	2243	4548	4548	9145	9146		
10^{-4}	486	481	2312	2312	4837	4818	10,364	10,319		
10^{-3}	486	481	2347	2366	5090	5088	11,190	11,177		
10^{-2}	486	481	2376	2404	5307	5326	11,958	12,006		
10^{-1}	486	481	2400	2417	5508	5517	12,702	12,738		
10^{0}	486	482	2408	2414	6242	6265				
10^{1}	486	482	2413	2416	6585	6595				
10^{2}	486	482	2416	2416	6955	6960				
10^{3}	486	483	2418	2419	7317	7328				

TABLE 13. COMPARISON OF VARIABLES AT JUNCTURE POINTS FOR THE CURVE FIT $T = T(p,\rho)$

Density ratio ρ/ρ_0	POINT A		POINT B		POINT C		POINT D		POINT E	
	lower	upper	lower	upper	lower	upper	lower	upper	lower	upper
10^{-7}	486	482	2089	2089	4025	4033	7864	7838		
10^{-6}	486	482	2165	2165	4281	4281	8470	8481		
10^{-5}	486	484	2242	2242	4549	4554	9146	9146		
10^{-4}	486	482	2310	2310	5064	5042	10,796	10,746		
10^{-3}	486	481	2363	2363	5386	5376	11,793	11,682		
10^{-2}	486	481	2404	2404	5690	5701	12,742	12,679		
10^{-1}	486	482	2402	2402	5968	5998	13,671	13,687		
10^{0}	486	482	2700	2700	6248	6267				
10^{1}	486	482	2706	2710	6585	6598				
10^{2}	486	483	2711	2712	6950	6959				
10^{3}	486	483	2713	2713	7309	7319				

TABLE 14. COMPARISON OF VARIABLES AT JUNCTURE POINTS FOR THE CURVE FIT $h = h(p,\rho)$

Density ratio ρ/ρ_o	POINT A		POINT B		POINT C		POINT D		POINT E	
	lower	upper	lower	upper	lower	upper	lower	upper	lower	upper
10^{-7}	$0.346x10^6$	$0.346x10^6$	$0.282x10^7$	$0.285x10^7$	$0.160x10^8$	$0.159x10^8$	$0.997x10^8$	$0.997x10^8$		
10^{-6}	$0.346x10^6$	$0.346x10^6$	$0.253x10^7$	$0.254x10^7$	$0.138x10^8$	$0.138x10^8$	$0.890x10^8$	$0.890x10^8$		
10^{-5}	$0.346x10^6$	$0.346x10^6$	$0.233x10^7$	$0.235x10^7$	$0.120x10^8$	$0.122x10^8$	$0.793x10^8$	$0.792x10^8$		
10^{-4}	$0.346x10^6$	$0.346x10^6$	$0.345x10^7$	$0.345x10^7$	$0.247x10^8$	$0.247x10^8$	$0.812x10^8$	$0.813x10^8$		
10^{-3}	$0.346x10^6$	$0.346x10^6$	$0.314x10^7$	$0.315x10^7$	$0.214x10^8$	$0.214x10^8$	$0.720x10^8$	$0.721x10^8$		
10^{-2}	$0.346x10^6$	$0.346x10^6$	$0.296x10^7$	$0.296x10^7$	$0.186x10^8$	$0.186x10^8$	$0.646x10^8$	$0.646x10^8$		
10^{-1}	$0.346x10^6$	$0.346x10^6$	$0.288x10^7$	$0.288x10^7$	$0.164x10^8$	$0.164x10^8$	$0.590x10^8$	$0.591x10^8$		
10^{0}	$0.345x10^6$	$0.345x10^6$	$0.386x10^7$	$0.387x10^7$	$0.201x10^8$	$0.202x10^8$				
10^{1}	$0.345x10^6$	$0.345x10^6$	$0.377x10^7$	$0.380x10^7$	$0.180x10^8$	$0.181x10^8$				
10^{2}	$0.345x10^6$	$0.345x10^6$	$0.374x10^7$	$0.376x10^7$	$0.166x10^8$	$0.166x10^8$				
10^{3}	$0.345x10^6$	$0.345x10^6$	$0.374x10^7$	$0.374x10^7$	$0.156x10^8$	$0.156x10^8$				

TABLE 15. COMPARISON OF COMPUTER TIMES

Curve Fit	Number of Data Points	Computer Time(seconds)		
		Old Subroutine	New Subroutine	NASA RGAS
$p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(e,\rho)$	10,661	0.54	0.77	1.86
$s = s(e,\rho)$	10,661	--	0.20	2.03
$T = T(p,\rho)$	9,921	0.25	0.31	0.84
$h = h(p,\rho)$	9,921	0.19	0.26	0.84
$\rho = \rho(p,s)$	3,038	--	0.10	1.07
$e = e(p,s)$	3,038	--	0.11	1.06
$a = a(p,s)$	3,038	--	0.11	1.06

previous TGAS subroutine which is 3.4 times faster. The new TGAS4 subroutine for $h = h(p,\rho)$ is 3.2 times faster than the NASA RGAS subroutine as compared to the previous TGAS subroutine which is 4.4 times faster. The TGAS2 subroutine for $s = s(e,\rho)$ was found to be 10.2 times faster than the NASA RGAS program. The subroutines TGAS5, TGAS6 and TGAS7 for the curve fits $\rho = \rho(p,s)$, $e = e(p,s)$ and $a = a(p,s)$, respectively, are approximately 10 times faster than the NASA RGAS subroutine. It should be noted that the NASA RGAS program requires two data files for storage of the cubic interpolation coefficients. The fact that these data files are now on disk and not tape has significantly speeded up the NASA RGAS subroutine. However, the curve fits still provide a substantial improvement in computing time, being 2.4 to 10.2 times faster than the table-lookup technique.

In conclusion, the new simplified curve fits for the thermodynamic properties of equilibrium air provide substantial reductions in computer time and storage while maintaining good accuracy. They can be incorporated into CFD computer codes in a straightforward manner without the need for data files. The improved accuracy of the new curve fits permits their use in "time-dependent" flow calculations from startup to the final "steady-state" solution. In addition, the improved continuity of these curve fits permits their use in iterative calculations. For example, the new curve fit for $h = h(p,\rho)$ can be employed in the iterative procedure required to "fit" a bow shock in equilibrium flow.

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APPENDIX

All the curve fits developed in this study have been incorporated into a single master program called TGAS. This master program permits the user to select the desired curve fit(s) from a "menu" of possibilities. The calling statement for this subroutine is

CALL TGAS (P,RHO,E,H,T,A,S,NTGAS)

with

P = pressure, N/m^2

RHO = density, kg/m^3

E = specific internal energy, m^2/s^2

H = specific enthalpy, m^2/s^2

T = temperature, K

A = speed of sound, m/s

S = specific entropy, $m^2/s^2 - K$

NTGAS = an integer flag to be set by the user for selection of the appropriate curve fit(s),

NTGAS = 0: $p = p(e,\rho)$

NTGAS = 1: $p = p(e,\rho)$, $a = a(e,\rho)$

NTGAS = 2: $p = p(e,\rho)$, $T = T(e,\rho)$

NTGAS = 3: $p = p(e,\rho)$, $a = a(e,\rho)$, $T = T(e,\rho)$

NTGAS = 4: $s = s(e,\rho)$

NTGAS = 5: $T = T(p,\rho)$

NTGAS = 6: $h = h(p,\rho)$

NTGAS = 7: $\rho = \rho(p,s)$

NTGAS = 8: $e = e(p,s)$

NTGAS = 9: $a = a(p,s)$

The curve fits for $p = p(e,\rho)$, $a = a(e,\rho)$ and $T = T(e,\rho)$ have been placed in a single subroutine TGAS1. Subroutine TGAS2 computes $s = s(e,\rho)$, subroutine TGAS3 computes $T = T(p,\rho)$, and subroutine TGAS4 computes $h = h(p,\rho)$. The curve fits $\rho = \rho(p,s)$, $e = e(p,s)$ and $a = a(p,s)$ have been placed in subroutines TGAS5, TGAS6 and TGAS7, respectively. The subroutines TGAS1 to TGAS7 can be used in a "stand-alone" manner if so desired, independent of the master program. A FORTRAN listing of these subroutines follows.

```
      SUBROUTINE TGAS(P,R,E,H,T,A,S,NTGAS)
C
C     PARAMETERS :
C
C     P = PRESSURE, IN NEWTONS/M**2
C     R = DENSITY, IN KG/M**3
C     E = SPECIFIC INTERNAL ENERGY, IN (M/SEC)**2
C     H = SPECIFIC ENTHALPY, IN (M/SEC)**2
C     T = TEMPERATURE, IN KELVIN
C     A = SPEED OF SOUND, IN M/SEC
C     S = SPECIFIC ENTROPY, IN (M/SEC)**2/KELVIN
C     NTGAS = INTEGER FLAG TO BE SET AS FOLLOWS,
C
C     NTGAS = 0 :
C     INPUT : R,E ; OUTPUT : P
C
C     NTGAS = 1 :
C     INPUT : R,E ; OUTPUT : P,A
C
C     NTGAS = 2 :
C     INPUT : R,E ; OUTPUT : P,T
C
C     NTGAS = 3 :
C     INPUT : R,E ; OUTPUT : P,A,T
C
C     NTGAS = 4 :
C     INPUT : R,E ; OUTPUT : S
C
C     NTGAS = 5 :
C     INPUT : P,R ; OUTPUT : T
C
C     NTGAS = 6 :
C     INPUT : P,R ; OUTPUT : H
C
C     NTGAS = 7 :
C     INPUT : P,S ; OUTPUT : R
C
C     NTGAS = 8 :
C     INPUT : P,S ; OUTPUT : E
C
C     NTGAS = 9 :
C     INPUT : P,S ; OUTPUT : A
C
      IF (NTGAS.EQ.0) CALL TGAS1(E,R,P,A,T,0)
      IF (NTGAS.EQ.1) CALL TGAS1(E,R,P,A,T,1)
      IF (NTGAS.EQ.2) CALL TGAS1(E,R,P,A,T,2)
      IF (NTGAS.EQ.3) CALL TGAS1(E,R,P,A,T,3)
      IF (NTGAS.EQ.4) CALL TGAS2(E,R,S)
      IF (NTGAS.EQ.5) CALL TGAS3(P,R,T)
      IF (NTGAS.EQ.6) CALL TGAS4(P,R,H)
```

```
IF (NTGAS.EQ.7) CALL TGAS5(P,S,R)
IF (NTGAS.EQ.8) CALL TGAS6(P,S,E)
IF (NTGAS.EQ.9) CALL TGAS7(P,S,A)
RETURN
END
```

```
      SUBROUTINE TGAS1 (E,R,P,A,T,MFLAG)
C
C     INPUTS FOR SUBROUTINE:
C
C     E= INTERNAL ENERGY, IN (M/SEC)**2 ;
C     R= DENSITY, IN KG/M**3 .
C
C     OUTPUTS:
C
C     P= PRESSURE, IN NEWTONS/M**2 ;
C     A= SPEED OF SOUND, IN M/SEC ;
C     T= TEMPERATURE, IN KELVIN .
C
C     IF MFLAG=0, RETURN P ;
C     IF MFLAG=1, RETURN P AND A ;
C     IF MFLAG=2, RETURN P AND T ;
C     IF MFLAG=3, RETURN P, A AND T .
C
      DATA E0,R0,P0,T0,GASCON/78408.4E00,1.292E00,1.0133E05,
     *273.15E00,287.06E00/
      RRATIO=R/R0
      ERATIO=E/E0
      Y=ALOG10(RRATIO)
      Z=ALOG10(ERATIO)
      LFLAG=0
      KFLAG=0
      IF (MFLAG.GT.1) LFLAG=1
      IF ((MFLAG.EQ.1).OR.(MFLAG.EQ.3)) KFLAG=1
      IF(ABS(Y+4.5E00).LT.2.5E-02) GO TO 20
      IF(ABS(Y+0.5E00).LT.5.0E-03) GO TO 50
      IFLAG=-1
      GO TO 90
 10   IF(LFLAG.EQ.1) GO TO 300
      RETURN
 20   IFLAG=0
      RSAVE=R
      YM=Y
      Y=-4.5E00+2.5E-02
      YHIGH=Y
      R=(10.**Y)*R0
      JFLAG=-1
      GO TO 90
 30   PHIGH=P
      AHIGH=A
      Y=-4.5E00-2.5E-02
      YLOW=Y
      R=(10.**Y)*R0
      JFLAG=0
      GO TO 90
 40   PLOW=P
```

```
      ALOW=A
      GO TO 80
50    IFLAG=1
      RSAVE=R
      YM=Y
      Y=-0.5E00+0.5E-02
      YHIGH=Y
      R=(10.**Y)*R0
      JFLAG=-1
      GO TO 90
60    PHIGH=P
      AHIGH=A
      Y=-0.5E00-0.5E-02
      YLOW=Y
      R=(10.**Y)*R0
      JFLAG=0
      GO TO 90
70    PLOW=P
      ALOW=A
80    P=PLOW+(PHIGH-PLOW)/(YHIGH-YLOW)*(YM-YLOW)
      A=ALOW+(AHIGH-ALOW)/(YHIGH-YLOW)*(YM-YLOW)
      R=RSAVE
      IF(LFLAG.EQ.1) GO TO 300
      RETURN
90    CONTINUE
      IF(Y.GT.-0.5E00) GO TO 200
      IF(Y.GT.-4.5E00) GO TO 150
      IF(Z.GT.0.65E00) GO TO 100
      GAMM=1.3965E00
      GO TO 250
100   IF(Z.GT.1.5E00) GO TO 110
      GAS1=1.52792E00-1.26953E-02*Y
      GAS2=(-6.13514E-01-5.08262E-02*Y)*Z
      GAS3=(-5.49384E-03+4.75120E-05*Z-3.18468E-04*Y)*Y*Y
      GAS4=(6.31835E-01+3.34012E-02*Y-2.19921E-01*Z)*Z*Z
      GAS5=-4.96286E01-1.17932E+01*Y
      GAS6=(6.91028E01+4.40405E+01*Y)*Z
      GAS7=(5.09249E00-1.40326E00*Z+2.08988E-01*Y)*Y*Y
      GAS8=(1.37308E01-1.78726E01*Y-1.86943E01*Z)*Z*Z
      GAS9=EXP(24.60452E00-2.E00*Y-2.093022E01*Z)
      DENO=1.-GAS9
      GAMM=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/DENO
      IF(KFLAG.EQ.0) GO TO 260
      GAS1R=-1.26953E-02
      GAS2R=-5.08262E-02*Z
      GAS3R=(-1.098768E-02-9.50240E-05*Z-9.554040E-04*Y)*Y
      GAS4R=3.34012E-02*Z*Z
      GAS5R=-1.17932E01
      GAS6R=4.40405E01*Z
      GAS7R=(1.018498E01-2.80652E00*Z+6.269641E-01*Y)*Y
```

```
      GAS8R=-1.78726E01*Z*Z
      GAS9R=-2.0E00
      GAS2E=GAS2/Z
      GAS3E=4.75120E-05*Y*Y
      GAS4E=(1.26367E00+6.68024E-02*Y-6.59763E-01*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=-1.40326E00*Y*Y
      GAS8E=(2.74616E01-3.57452E01*Y-5.60829E01*Z)*Z
      GAS9E=-2.093022E01
      GAMMR=GAS1R+GAS2R+GAS3R+GAS4R+(GAS5R+GAS6R+GAS7R+GAS8R)/DENO
     *+(GAS5+GAS6+GAS7+GAS8)*GAS9R*GAS9/(DENO**2)
      GAMME=GAS2E+GAS3E+GAS4E+(GAS6E+GAS7E+GAS8E)/DENO
     *+(GAS5+GAS6+GAS7+GAS8)*GAS9E*GAS9/(DENO**2)
      GO TO 260
110   IF(Z.GT.2.2E00) GO TO 120
      GAS1=-1.70333E01-5.08545E-01*Y
      GAS2=(2.46299E01+4.45617E-01*Y)*Z
      GAS3=(-8.95298E-03+2.29618E-03*Z-2.89186E-04*Y)*Y*Y
      GAS4=(-1.10204E01-9.89727E-02*Y+1.62903E00*Z)*Z*Z
      GAS5=1.86797E01+5.19662E-01*Y
      GAS6=(-2.41338E01-4.34837E-01*Y)*Z
      GAS7=(9.16089E-03-1.52082E-03*Z+3.46482E-04*Y)*Y*Y
      GAS8=(1.02035E01+9.70762E-02*Y-1.39460E00*Z)*Z*Z
      GAS9=(-1.42762E02-1.647088E00*Y+7.660312E01*Z
     *+8.259346E-01*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-5.08545E-01
      GAS2R=4.45617E-01*Z
      GAS3R=(-1.790596E-02+4.59236E-03*Z-8.67558E-04*Y)*Y
      GAS4R=-9.89727E-02*Z*Z
      GAS5R=5.19662E-01
      GAS6R=-4.34837E-01*Z
      GAS7R=(1.832178E-02-3.04164E-03*Z+1.039446E-03*Y)*Y
      GAS8R=9.70762E-02*Z*Z
      GAS9R=-1.647088E00+8.259346E-01*Z
      GAS2E=GAS2/Z
      GAS3E=2.29618E-03*Y*Y
      GAS4E=(-2.20408E01-1.979454E-01*Y+4.88709E00*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=-1.52082E-03*Y*Y
      GAS8E=(2.0407E01+1.941524E-01*Y-4.1838E00*Z)*Z
      GAS9E=7.660312E01+8.259346E-01*Y
      GO TO 240
120   IF (Z.GT.3.05E00) GO TO 130
      GAS1=2.24374E00+1.03073E-01*Y
      GAS2=(-5.32238E-01-5.59852E-02*Y)*Z
      GAS3=(3.56484E-03-1.01359E-04*Z+1.59127E-04*Y)*Y*Y
      GAS4=(-4.80156E-02+1.06794E-02*Y+3.66035E-02*Z)*Z*Z
      GAS5=-5.70378E00-3.10056E-01*Y
      GAS6=(5.01094E00+1.80411E-01*Y)*Z
```

```
      GAS7=(-9.49361E-03+1.94839E-03*Z-2.24908E-04*Y)*Y*Y
      GAS8=(-1.40331E00-2.79718E-02*Y+1.20278E-01*Z)*Z*Z
      GAS9=(1.139755E02-4.985467E00*Y-4.223833E01*Z
     *+2.009706E00*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=1.03073E-01
      GAS2R=-5.59852E-02*Z
      GAS3R=(7.12968E-03-2.0218E-04*Z+4.77381E-04*Y)*Y
      GAS4R=1.06794E-02*Z*Z
      GAS5R=-3.10056E-01
      GAS6R=1.80411E-01*Z
      GAS7R=(-1.898722E-02+3.89678E-03*Z-6.74724E-04*Y)*Y
      GAS8R=-2.79718E-02*Z*Z
      GAS9R=-4.985467E00+2.009706E00*Z
      GAS2E=GAS2/Z
      GAS3E=-1.01359E-04*Y*Y
      GAS4E=(-9.60312E-02+2.13588E-02*Y+1.098105E-01*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=1.94839E-03*Y*Y
      GAS8E=(-2.80662E00-5.59436E-02*Y+3.60834E-01*Z)*Z
      GAS9E=-4.223833E01+2.009706E00*Y
      GO TO 240
130   IF(Z.GT.3.4E00) GO TO 140
      GAS1=-0.20807E02+0.40197E00*Y
      GAS2=(0.22591E02-0.25660E00*Y)*Z
      GAS3=(-0.95833E-03+0.23966E-02*Z+0.33671E-03*Y)*Y*Y
      GAS4=(-0.77174E01+0.4606E-01*Y+0.878E00*Z)*Z*Z
      GAS5=-0.21737E03-0.46927E01*Y
      GAS6=(0.18101E03+0.26621E01*Y)*Z
      GAS7=(-0.34759E-01+0.64681E-02*Z-0.70391E-03*Y)*Y*Y
      GAS8=(-0.50019E02-0.38381E00*Y+0.45795E01*Z)*Z*Z
      GAS9=(0.4544373E03+0.1250133E02*Y-0.1376001E03*Z
     *-0.3641774E01*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=0.40197E00
      GAS2R=-0.25660E00*Z
      GAS3R=(-1.91666E-03+4.7932E-03*Z+1.01013E-03*Y)*Y
      GAS4R=0.4606E-01*Z*Z
      GAS5R=-0.46927E01
      GAS6R=0.26621E01*Z
      GAS7R=(-6.9518E-02+1.29362E-02*Z-2.11173E-03*Y)*Y
      GAS8R=-0.38381E00*Z*Z
      GAS9R=0.1250133E02-0.3641774E01*Z
      GAS2E=GAS2/Z
      GAS3E=0.23966E-02*Y*Y
      GAS4E=(-1.54348E01+9.212E-02*Y+2.634E00*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=0.64681E-02*Y*Y
      GAS8E=(-1.00038E02-7.6762E-01*Y+1.37385E01*Z)*Z
      GAS9E=-0.1376001E03-0.3641774E01*Y
```

```
      GO TO 240
140   IF(Z.GT.3.69E00) WRITE(6,1000) R,E
      GAS1=-5.22951E01-4.00011E-01*Y
      GAS2=(4.56439E01+2.24484E-01*Y)*Z
      GAS3=(-3.73775E-03+2.43161E-03*Z+2.24755E-04*Y)*Y*Y
      GAS4=(-1.29756E01-2.79517E-02*Y+1.22998E00*Z)*Z*Z
      GAMM=GAS1+GAS2+GAS3+GAS4
      IF(KFLAG.EQ.0) GO TO 260
      GAS1R=-4.00011E-01
      GAS2R=2.24484E-01*Z
      GAS3R=(-7.4755E-03+4.86322E-03*Z+6.74265E-04*Y)*Y
      GAS4R=-2.79517E-02*Z*Z
      GAS2E=GAS2/Z
      GAS3E=2.43161E-03*Y*Y
      GAS4E=(-2.59512E01-5.59034E-02*Y+3.68994E00*Z)*Z
      GAMMR=GAS1R+GAS2R+GAS3R+GAS4R
      GAMME=GAS2E+GAS3E+GAS4E
      GO TO 260
150   IF(Z.GT.0.65E00) GO TO 160
      GAMM=1.398E00
      GO TO 250
160   IF (Z.GT.1.5E00) GO TO 170
      GAS1=1.39123E00-4.08321E-03*Y
      GAS2=(1.42545E-02+1.41769E-02*Y)*Z
      GAS3=(2.57225E-04+6.52912E-04*Z+8.46912E-05*Y)*Y*Y
      GAS4=(6.2555E-02-7.83637E-03*Y-9.78720E-02*Z)*Z*Z
      GAS5=5.80955-1.82302E-01*Y
      GAS6=(-9.62396E00+1.79619E-01*Y)*Z
      GAS7=(-2.30518E-02+1.18720E-02*Z-3.35499E-04*Y)*Y*Y
      GAS8=(5.27047E00-3.65507E-02*Y-9.19897E-01*Z)*Z*Z
      GAS9=(-10.0E00*Z+14.2E00)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-4.08321E-03
      GAS2R=1.41769E-02*Z
      GAS3R=(5.1445E-04+1.305824E-03*Z+2.540736E-04*Y)*Y
      GAS4R=-7.83637E-03*Z*Z
      GAS5R=-1.82302E-01
      GAS6R=1.79619E-01*Z
      GAS7R=(-4.61036E-02+2.3744E-02*Z-1.006497E-03*Y)*Y
      GAS8R=-3.65507E-02*Z*Z
      GAS9R=0.0E00
      GAS2E=GAS2/Z
      GAS3E=6.52912E-04*Y*Y
      GAS4E=(1.2511E-01-1.567274E-02*Y-2.93616E-01*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=1.1872E-02*Y*Y
      GAS8E=(1.054094E01-7.31014E-02*Y-2.759691E00*Z)*Z
      GAS9E=-10.0E00
      GO TO 240
170   IF (Z.GT.2.22E00) GO TO 180
```

```
      GAS1=-1.20784E00-2.57909E-01*Y
      GAS2=(5.02307E00+2.87201E-01*Y)*Z
      GAS3=(-9.95577E-03+5.23524E-03*Z-1.45574E-04*Y)*Y*Y
      GAS4=(-3.20619E00-7.50405E-02*Y+6.51564E-01*Z)*Z*Z
      GAS5=-6.62841E00+2.77112E-02*Y
      GAS6=(7.30762E00-7.68230E-02*Y)*Z
      GAS7=(7.19421E-03-3.62463E-03*Z+1.62777E-04*Y)*Y*Y
      GAS8=(-2.33161E00+3.04767E-02*Y+1.66856E-01*Z)*Z*Z
      GAS9=(1.255324E02+2.015335E00*Y-6.390747E01*Z-
     *6.515225E-01*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-2.57909E-01
      GAS2R=2.87201E-01*Z
      GAS3R=(-1.991154E-02+1.047048E-02*Z-4.36722E-04*Y)*Y
      GAS4R=-7.50405E-02*Z*Z
      GAS5R=2.77112E-02
      GAS6R=-7.6823E-02*Z
      GAS7R=(1.438842E-02-7.24926E-03*Z+4.88331E-04*Y)*Y
      GAS8R=3.04767E-02*Z*Z
      GAS9R=2.015335E00-6.515225E-01*Z
      GAS2E=GAS2/Z
      GAS3E=5.23524E-03*Y*Y
      GAS4E=(-6.41238E00-1.50081E-01*Y+1.954692E00*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=-3.62463E-03*Y*Y
      GAS8E=(-4.66322E00+6.09534E-02*Y+5.00568E-01*Z)*Z
      GAS9E=-6.390747E01-6.515225E-01*Y
      GO TO 240
180   IF (Z.GT.2.95) GO TO 190
      GAS1=-2.26460E00-7.82263E-02*Y
      GAS2=(4.90497E00+7.18096E-02*Y)*Z
      GAS3=(-3.06443E-03+1.74209E-03*Z+2.84214E-05*Y)*Y*Y
      GAS4=(-2.24750E00-1.31641E-02*Y+3.33658E-01*Z)*Z*Z
      GAS5=-1.47904E01-1.76627E-01*Y
      GAS6=(1.35036E01+8.77280E-02*Y)*Z
      GAS7=(-2.13327E-03+7.15487E-04*Z+7.30928E-05*Y)*Y*Y
      GAS8=(-3.95372E00-8.96151E-03*Y+3.63229E-01*Z)*Z*Z
      GAS9=(1.788542E02+6.317894E00*Y-6.756741E01*Z-
     *2.460060E00*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-7.82263E-02
      GAS2R=7.18096E-02*Z
      GAS3R=(-6.12886E-03+3.48418E-03*Z+8.52642E-05*Y)*Y
      GAS4R=-1.31641E-02*Z*Z
      GAS5R=-1.76627E-01
      GAS6R=8.7728E-02*Z
      GAS7R=(-4.26654E-03+1.430974E-03*Z+2.192784E-04*Y)*Y
      GAS8R=-8.96151E-03*Z*Z
      GAS9R=6.317894E00-2.46006E00*Z
      GAS2E=GAS2/Z
```

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      GAS3E=1.74209E-03*Y*Y
      GAS4E=(-4.495E00-2.63282E-02*Y+1.000974E00*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=7.15487E-04*Y*Y
      GAS8E=(-7.90744E00-1.792302E-02*Y+1.089687E00*Z)*Z
      GAS9E=-6.756741E01-2.46006E00*Y
      GO TO 240
190   IF(Z.GT.3.4E00) WRITE(6,1000) R,E
      GAS1=-1.66904E01-2.58318E-01*Y
      GAS2=(1.78350E01+1.54898E-01*Y)*Z
      GAS3=(-9.71263E-03+3.97740E-03*Z+9.04300E-05*Y)*Y*Y
      GAS4=(-5.94108E00-2.01335E-02*Y+6.60432E-01*Z)*Z*Z
      GAS5=8.54690E01+1.17554E01*Y
      GAS6=(-7.21760E01-7.15723E00*Y)*Z
      GAS7=(-4.16150E-02+1.38147E-02*Z+5.45184E-04*Y)*Y*Y
      GAS8=(2.01758E01+1.08990E00*Y-1.86438E00*Z)*Z*Z
      GAS9=(2.883262E02+1.248536E01*Y-8.816985E01*Z-
     *3.720309E00*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-2.58318E-01
      GAS2R=1.54898E-01*Z
      GAS3R=(-1.942526E-02+7.9548E-03*Z+2.7129E-04*Y)*Y
      GAS4R=-2.01335E-02*Z*Z
      GAS5R=1.17554E01
      GAS6R=-7.15723E00*Z
      GAS7R=(-8.323E-02+2.76294E-02*Z+1.635552E-03*Y)*Y
      GAS8R=1.0899E00*Z*Z
      GAS9R=1.248536E01-3.720309E00*Z
      GAS2E=GAS2/Z
      GAS3E=3.9774E-03*Y*Y
      GAS4E=(-1.188216E01-4.0267E-02*Y+1.981296E00*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=1.38147E-02*Y*Y
      GAS8E=(4.03516E01+2.1798E00*Y-5.59314E00*Z)*Z
      GAS9E=-8.816985E01-3.720309E00*Y
      GO TO 240
200   IF(Z.GT.0.65E00) GO TO 210
      GAMM=1.3988E00
      GO TO 250
210   IF(Z.GT.1.7E00) GO TO 220
      GAS1=1.37062E00+1.29673E-02*Y
      GAS2=(1.11418E-01-3.26912E-02*Y)*Z
      GAS3=(1.06869E-03-2.00286E-03*Z+2.38305E-04*Y)*Y*Y
      GAS4=(-1.06133E-01+1.90251E-02*Y+3.02210E-03*Z)*Z*Z
      GAMM=GAS1+GAS2+GAS3+GAS4
      IF(KFLAG.EQ.0) GO TO 260
      GAS1R=1.29673E-02
      GAS2R=-3.26912E-02*Z
      GAS3R=(2.13738E-03-4.00572E-03*Z+7.14915E-04*Y)*Y
      GAS4R=1.90251E-02*Z*Z
```

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      GAS2E=GAS2/Z
      GAS3E=-2.00286E-03*Y*Y
      GAS4E=(-2.12266E-01+3.80502E-02*Y+9.0663E-03*Z)*Z
      GAMMR=GAS1R+GAS2R+GAS3R+GAS4R
      GAMME=GAS2E+GAS3E+GAS4E
      GO TO 260
220   IF(Z.GT.2.35) GO TO 230
      GAS1=3.43846E-02-2.33584E-01*Y
      GAS2=(2.85574E00+2.59787E-01*Y)*Z
      GAS3=(-10.89927E-03+4.23659E-03*Z+3.85712E-04*Y)*Y*Y
      GAS4=(-1.94785E00-6.73865E-02*Y+4.08518E-01*Z)*Z*Z
      GAS5=-4.20569E00+1.33139E-01*Y
      GAS6=(4.51236E00-1.66341E-01*Y)*Z
      GAS7=(1.67787E-03-1.10022E-03*Z+3.06676E-04*Y)*Y*Y
      GAS8=(-1.35516E00+4.91716E-02*Y+7.52509E-02*Z)*Z*Z
      GAS9=(1.757042E02-2.163278E00*Y-8.833702E01*Z+
     *1.897543E00*Y*Z)
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-2.33584E-01
      GAS2R=2.59787E-01*Z
      GAS3R=(-2.179854E-02+8.47318E-03*Z+1.157136E-03*Y)*Y
      GAS4R=-6.73865E-02*Z*Z
      GAS5R=1.33139E-01
      GAS6R=-1.66341E-01*Z
      GAS7R=(3.35574E-03-2.20044E-03*Z+9.20028E-04*Y)*Y
      GAS8R=4.91716E-02*Z*Z
      GAS9R=-2.163278E00+1.897543E00*Z
      GAS2E=GAS2/Z
      GAS3E=4.23659E-03*Y*Y
      GAS4E=(-3.8957E00-1.34773E-01*Y+1.225554E00*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=-1.10022E-03*Y*Y
      GAS8E=(-2.71032E00+9.83432E-02*Y+2.257527E-01*Z)*Z
      GAS9E=-8.833702E01+1.897543E00*Y
      GO TO 240
230   IF(Z.GT.2.9E00) WRITE(6,1000) R,E
      GAS1=-1.70633E00-1.48403E-01*Y
      GAS2=(4.23104E00+1.37290E-01*Y)*Z
      GAS3=(-9.10934E-03+3.85707E-03*Z+2.69026E-04*Y)*Y*Y
      GAS4=(-1.97292E00-2.81830E-02*Y+2.95882E-01*Z)*Z*Z
      GAS5=3.41580E01-1.89972E01*Y
      GAS6=(-4.0858E01+1.30321E01*Y)*Z
      GAS7=(-8.01272E-01+2.75121E-01*Z-1.77969E-04*Y)*Y*Y
      GAS8=(1.60826E01-2.23386E00*Y-2.08853E00*Z)*Z*Z
      GAS9=(2.561323E02+1.737089E02*Y-9.058890E01*Z-
     *5.838803E01*Y*Z)
      IF(GAS9.GT.30.E00) GAS9=30.E00
      IF(GAS9.LT.-30.E00) GAS9=-30.E00
      IF(KFLAG.EQ.0) GO TO 240
      GAS1R=-1.48403E-01
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      GAS2R=1.3729E-01*Z
      GAS3R=(-1.821868E-02+7.71414E-03*Z+8.07078E-04*Y)*Y
      GAS4R=-2.8183E-02*Z*Z
      GAS5R=-1.89972E01
      GAS6R=1.30321E01*Z
      GAS7R=(-1.602544E00+5.50242E-01*Z-5.33907E-04*Y)*Y
      GAS8R=-2.23386E00*Z*Z
      GAS9R=1.737089E02-5.838803E01*Z
      GAS2E=GAS2/Z
      GAS3E=3.85707E-03*Y*Y
      GAS4E=(-3.94584E00-5.6366E-02*Y+8.87646E-01*Z)*Z
      GAS6E=GAS6/Z
      GAS7E=2.75121E-01*Y*Y
      GAS8E=(3.21652E01-4.46772E00*Y-6.26559E00*Z)*Z
      GAS9E=-9.05889E01-5.838803E01*Y
240   GAS9=EXP(GAS9)
      GAMM=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
      IF(KFLAG.EQ.0) GO TO 260
      GAMMR=GAS1R+GAS2R+GAS3R+GAS4R+(GAS5R+GAS6R+GAS7R+GAS8R)/
     *(1.+GAS9)-(GAS5+GAS6+GAS7+GAS8)*GAS9R*GAS9/((1.+GAS9)**2)
      GAMME=GAS2E+GAS3E+GAS4E+(GAS6E+GAS7E+GAS8E)/(1.+GAS9)
     *-(GAS5+GAS6+GAS7+GAS8)*GAS9E*GAS9/((1.+GAS9)**2)
      GO TO 260
250   CONTINUE
      IF(KFLAG.EQ.0) GO TO 260
      GAMMR=0.0E00
      GAMME=0.0E00
260   P=(GAMM-1)*E*R
      IF(KFLAG.EQ.0) GO TO 270
      GAMMR=GAMMR/2.302585E00
      GAMME=GAMME/2.302585E00
      ASQ=E*((GAMM-1.E00)*(GAMM+GAMME)+GAMMR)
      A=SQRT(ASQ)
270   IF(IFLAG) 10,280,290
280   IF(JFLAG) 30,40,10
290   IF(JFLAG) 60,70,10
300   X=ALOG10(P/P0)
      Y=ALOG10(R/R0)
      Z1=X-Y
      IF (Y.GT.-0.5E00) GO TO 400
      IF(Y.GT.-4.5E00) GO TO 350
      IF(Z1.GT.0.25E00) GO TO 310
      T=P/(GASCON*R)
      RETURN
310   IF(Z1.GT.0.95E00) GO TO 320
      GAS1=1.44824E-01+1.36744E-02*Y
      GAS2=(1.17099E-01-8.22299E-02*Y)*Z1
      GAS3=(-6.75303E-04-1.47314E-03*Z1-7.90851E-05*Y)*Y*Y
      GAS4=(1.3937E00+6.83066E-02*Y-6.65673E-01*Z1)*Z1*Z1
      TNON=GAS1+GAS2+GAS3+GAS4
```

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      GO TO 450
320   IF(Z1.GT.1.4E00) GO TO 330
      GAS1=-9.325E00-9.32017E-01*Y
      GAS2=(2.57176E01+1.61292E00*Y)*Z1
      GAS3=(-3.00242E-02+2.62959E-02*Z1-2.77651E-04*Y)*Y*Y
      GAS4=(-2.1662E01-6.81431E-01*Y+6.26962E00*Z1)*Z1*Z1
      GAS5=-3.38534E00+1.82594E-01*Y
      GAS6=(1.84928E-01-7.01109E-01*Y)*Z1
      GAS7=(1.10150E-02-1.60570E-02*Z1+1.57701E-05*Y)*Y*Y
      GAS8=(5.4702E00+4.11624E-01*Y-2.81498E00*Z1)*Z1*Z1
      GAS9=EXP(-3.887015E01-2.908228E01*Y+4.070557E01*Z1
     *+2.682347E01*Y*Z1)
      GO TO 440
330   IF( Z1.GT.1.95E00) GO TO 340
      GAS1=-1.93082E01-1.54557E00*Y
      GAS2=(3.69035E01+1.92214E00*Y)*Z1
      GAS3=(-3.59027E-02+2.31827E-02*Z1-2.01327E-04*Y)*Y*Y
      GAS4=(-2.20440E01-5.80935E-01*Y+4.43367E00*Z1)*Z1*Z1
      GAS5=-3.83069E00+1.32864E-01*Y
      GAS6=(-3.91902E00-6.79564E-01*Y)*Z1
      GAS7=(6.06341E-04-8.12997E-03*Z1-1.61012E-04*Y)*Y*Y
      GAS8=(7.24632E00+3.15461E-01*Y-2.17879E00*Z1)*Z1*Z1
      GAS9=EXP(2.08E01-2.56E01*Y+1.0E00*Z1+1.80E01*Y*Z1)
      GO TO 440
340   GAS1=-2.59721E01-1.77419E00*Y
      GAS2=(3.62495E01+1.55383E00*Y)*Z1
      GAS3=(-4.51359E-02+2.43648E-02*Z1+1.2804E-04*Y)*Y*Y
      GAS4=(-1.59988E01-3.17807E-01*Y+2.40584E00*Z1)*Z1*Z1
      GAS5=-1.81433E01+1.54896E-01*Y
      GAS6=(1.26582E01-3.66275E-01*Y)*Z1
      GAS7=(3.24496E-02-1.66385E-02*Z1+3.02177E-04*Y)*Y*Y
      GAS8=(-1.41759E00+1.11241E-01*Y-3.10983E-01*Z1)*Z1*Z1
      GAS9=EXP(1.115884E02-6.452606E00*Y-5.337863E01*Z1
     *+2.026986E00*Y*Z1)
      GO TO 440
350   IF(Z1.GT.0.25E00) GO TO 360
      T=P/(GASCON*R)
      RETURN
360   IF(Z1.GT.0.95E00) GO TO 370
      GAS1=2.94996E-02+7.24997E-03*Y
      GAS2=(7.81783E-01-3.27402E-02*Y)*Z1
      GAS3=(3.23357E-04-9.69989E-04*Z1-8.93240E-06*Y)*Y*Y
      GAS4=(3.95198E-01+2.92926E-02*Y-2.12182E-01*Z1)*Z1*Z1
      TNON=GAS1+GAS2+GAS3+GAS4
      GO TO 450
370   IF (Z1.GT.1.4E00) GO TO 380
      GAS1=-5.53324E00-3.53749E-01*Y
      GAS2=(1.63638E01+5.87547E-01*Y)*Z1
      GAS3=(-1.16081E-02+7.99571E-03*Z1-2.79316E-04*Y)*Y*Y
      GAS4=(-1.41239E01-2.35146E-01*Y+4.28891E00*Z1)*Z1*Z1
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      GAS5=9.07979E00+1.01308E00*Y
      GAS6=(-2.29428E01-1.52122E00*Y)*Z1
      GAS7=(3.78390E-02-2.63115E-02*Z1+5.46402E-04*Y)*Y*Y
      GAS8=(1.95657E01+5.73839E-01*Y-5.63057E00*Z1)*Z1*Z1
      GAS9=EXP(7.619803E01-1.501155E01*Y-6.770845E01*Z1
     *+1.273147E01*Y*Z1)
      GO TO 440
380   IF (Z1.GT.2.00E00) GO TO 390
      GAS1=-1.13598E01-1.02049E00*Y
      GAS2=(2.22793E01+1.24038E00*Y)*Z1
      GAS3=(-3.10771E-02+1.92551E-02*Z1-2.69140E-04*Y)*Y*Y
      GAS4=(-1.31512E01-3.62875E-01*Y+2.64544E00*Z1)*Z1*Z1
      GAS5=8.72852E00+1.27564E00*Y
      GAS6=(-1.79172E01-1.52051E00*Y)*Z1
      GAS7=(4.91264E-02-2.81731E-02*Z1+5.23383E-04*Y)*Y*Y
      GAS8=(1.16719E01+4.45413E-01*Y-2.45584E00*Z1)*Z1*Z1
      GAS9=EXP(1.84792E02+9.583443E00*Y-1.020835E02*Z1
     *-4.166727E00*Y*Z1)
      GO TO 440
390   GAS1=-1.76079E01-1.26579E00*Y
      GAS2=(2.48544E01+1.09442E00*Y)*Z1
      GAS3=(-3.65534E-02+1.54346E-02*Z1-4.59822E-04*Y)*Y*Y
      GAS4=(-1.08166E01-2.27803E-01*Y+1.60641E00*Z1)*Z1*Z1
      GAS5=2.60669E01+2.31791E00*Y
      GAS6=(-3.22433E01-1.82645E00*Y)*Z1
      GAS7=(4.94621E-02-1.85542E-02*Z1+5.04815E-04*Y)*Y*Y
      GAS8=(1.33829E01+3.59744E-01*Y-1.86517E00*Z1)*Z1*Z1
      GAS9=EXP(3.093755E02+1.875018E01*Y-1.375004E02*Z1
     *-8.333418E00*Y*Z1)
      GO TO 440
400   IF (Z1.GT.0.25E00) GO TO 410
      T=P/(GASCON*R)
      RETURN
410   IF (Z1.GT.0.95E00) GO TO 420
      GAS1=-2.94081E-03+5.73915E-04*Y
      GAS2=(9.88883E-01-3.71241E-03*Y)*Z1
      GAS3=(1.12387E-04-3.76528E-04*Z1+1.76192E-05*Y)*Y*Y
      GAS4=(2.86656E-02+4.56059E-03*Y-1.99498E-02*Z1)*Z1*Z1
      TNON=GAS1+GAS2+GAS3+GAS4
      GO TO 450
420   IF (Z1.GT.1.45E00) GO TO 430
      GAS1=1.32396E00+8.52771E-02*Y
      GAS2=(-3.24257E00-2.00937E-01*Y)*Z1
      GAS3=(5.68146E-03-6.85856E-03*Z1+1.98366E-04*Y)*Y*Y
      GAS4=(4.53823E00+1.18123E-01*Y-1.6246E00*Z1)*Z1*Z1
      GAS5=-5.26673E-01-1.58691E-01*Y
      GAS6=(2.61600E00+3.16356E-01*Y)*Z1
      GAS7=(-1.90755E-02+1.70124E-02*Z1-5.58398E-04*Y)*Y*Y
      GAS8=(-3.3793E00-1.52212E-01*Y+1.30757E00*Z1)*Z1*Z1
      GAS9=EXP(1.442206E02-2.544727E01*Y-1.277055E02*Z1
```

```
     *+2.236647E01*Y*Z1)
      GO TO 440
430   GAS1=-1.60643E00-5.07368E-02*Y
      GAS2=(3.95872E00+3.69383E-02*Y)*Z1
      GAS3=(-1.59378E-03+1.06057E-03*Z1+6.53278E-05*Y)*Y*Y
      GAS4=(-1.71201E00+9.25124E-03*Y+2.71039E-01*Z1)*Z1*Z1
      GAS5=1.80476E01+1.62964E00*Y
      GAS6=(-2.73124E01-1.57430E00*Y)*Z1
      GAS7=(5.85277E-02-2.77313E-02*Z1+1.16146E-03*Y)*Y*Y
      GAS8=(1.36342E01+3.70714E-01*Y-2.23787E00*Z1)*Z1*Z1
      GAS9=EXP(1.292515E02+1.360552E00*Y-7.07482E01*Z1
     *+1.360532E00*Y*Z1)
440   TNON=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
450   T=(10.**TNON)*T0
1000  FORMAT(/20X,48HWARNING!  OUTSIDE OF VALIDITY RANGE OF CURVE FIT
     *,/,20X,5HRHO =,1PE15.8,5X,3HE =,1PE15.8,/)
      RETURN
      END
```

```
      SUBROUTINE TGAS2(E,R,S)
C
C     INPUTS FOR SUBROUTINE:
C     E=INTERNAL ENERGY IN (M/SEC)**2
C     R=DENSITY IN KG/M**3
C
C     OUTPUT:
C     S=ENTROPY IN (M/SEC)**2/K
C
C
      DATA E0,R0,GASCON/78408.4E00,1.292E00,287.06E00/
      RRATIO=R/R0
      ERATIO=E/E0
      Y=ALOG10(RRATIO)
      Z=ALOG10(ERATIO)
      IF(ABS(Y+4.5E00).LT.2.5E-02) GO TO 10
      IF(ABS(Y+0.5E00).LT.0.5E-02) GO TO 40
      IFLAG=-1
      GO TO 80
 10   IFLAG=0
      RSAVE=R
      YM=Y
      Y=-4.5E00+2.5E-02
      YHIGH=Y
      R=(10.**Y)*R0
      JFLAG=-1
      GO TO 80
 20   SHIGH=S
      Y=-4.5E00-2.5E-02
      YLOW=Y
      R=(10.**Y)*R0
      JFLAG=0
      GO TO 80
 30   SLOW=S
      GO TO 70
 40   IFLAG=1
      RSAVE=R
      YM=Y
      Y=-0.5E00+0.5E-02
      YHIGH=Y
      R=(10.**Y)*R0
      JFLAG=-1
      GO TO 80
 50   SHIGH=S
      Y=-0.5E00-0.5E-02
      YLOW=Y
      R=(10.**Y)*R0
      JFLAG=0
      GO TO 80
 60   SLOW=S
```

```
70     S=SLOW+(SHIGH-SLOW)/(YHIGH-YLOW)*(YM-YLOW)
       R=RSAVE
       RETURN
80     CONTINUE
       IF(Z.LE.0.65E00) GO TO 110
       IF(Y.GT.-4.5E00) GO TO 90
       IF(Z.GT.3.69E00) WRITE(6,1000) R,E
       GAS1=-9.91081E-01-5.00277E00*Y
       GAS2=(5.46521E01+5.10144E00*Y)*Z
       GAS3=(1.76206E-02+2.12002E-02*Z+1.76358E-03*Y)*Y*Y
       GAS4=(-2.97001E01-1.84915E00*Y+5.87892E00*Z)*Z*Z
       GO TO 120
90     IF(Y.GT.-0.50E00) GO TO 100
       IF(Z.GT.3.4E00) WRITE(6,1000) R,E
       GAS1=1.0836E01-4.55524E00*Y
       GAS2=(2.96473E01+3.90851E00*Y)*Z
       GAS3=(-2.05732E-03+3.65982E-02*Z+5.23821E-03*Y)*Y*Y
       GAS4=(-1.67001E01-1.44623E00*Y+3.98307E00*Z)*Z*Z
       GO TO 120
100    IF(Z.GT.3.0E00) WRITE(6,1000) R,E
       GAS1=2.01858E01-3.13458E00*Y
       GAS2=(1.03619E01+1.87767E00*Y)*Z
       GAS3=(-1.72922E-01+1.12174E-01*Z+1.28626E-02*Y)*Y*Y
       GAS4=(-5.43557E00-8.71048E-01*Y+2.01789E00*Z)*Z*Z
       GO TO 120
110    DELTZ=Z-0.4E00
       DELTS=(2.5E00*DELTZ-Y)*GASCON*2.302585E00
       S=6779.2004E00+DELTS
       GO TO 130
120    SNON=GAS1+GAS2+GAS3+GAS4
       S=GASCON*SNON
130    IF(IFLAG) 160,140,150
140    IF(JFLAG) 20,30,160
150    IF(JFLAG) 50,60,160
160    CONTINUE
1000   FORMAT(/20X,48HWARNING!  OUTSIDE VALIDITY RANGE OF CURVE FIT
      *,/,20X,5HRHO =,1PE15.8,5X,3HE =,1PE15.8,/)
       RETURN
       END
```

```
      SUBROUTINE TGAS3 (P,RHO,T)
C
C     INPUTS FOR SUBROUTINE :
C
C     P= PRESSURE, IN NEWTONS/M**2.
C     RHO= DENSITY, IN KG/M**3.
C
C     OUTPUT :
C
C     T= TEMPERATURE, IN KELVIN.
C
C
      DATA R0,P0,T0,GASCON/1.292E00,1.0133E05,273.15E00,287.06E00/
      Y=ALOG10(RHO/R0)
      X=ALOG10(P/P0)
      IF(ABS(Y+4.5E00).LT.2.5E-02) GO TO 20
      IF(ABS(Y+0.5E00).LT.5.0E-03) GO TO 50
      IFLAG=-1
      GO TO 90
 10   RETURN
 20   IFLAG=0
      RSAVE=RHO
      YM=Y
      Y=-4.5E00+2.5E-02
      YHIGH=Y
      RHO=(10.**Y)*R0
      JFLAG=-1
      GO TO 90
 30   THIGH=T
      Y=-4.5E00-2.5E-02
      YLOW=Y
      RHO=(10.**Y)*R0
      JFLAG=0
      GO TO 90
 40   TLOW=T
      GO TO 80
 50   IFLAG=1
      RSAVE=RHO
      YM=Y
      Y=-0.5E00+0.5E-02
      YHIGH=Y
      RHO=(10.**Y)*R0
      JFLAG=-1
      GO TO 90
 60   THIGH=T
      Y=-0.5E00-0.5E-02
      YLOW=Y
      RHO=(10.**Y)*R0
      JFLAG=0
      GO TO 90
```

```
70      TLOW=T
80      T=TLOW+(THIGH-TLOW)/(YHIGH-YLOW)*(YM-YLOW)
        RHO=RSAVE
        RETURN
90      Z1=X-Y
        IF (Y.GT.-0.5E00) GO TO 190
        IF (Y.GT.-4.5E00) GO TO 140
        IF (Z1.GT.0.25E00) GO TO 100
        T=P/(RHO*GASCON)
        GO TO 250
100     IF (Z1.GT.0.95E00) GO TO 110
        GAS1=1.23718E-01+1.08623E-02*Y
        GAS2=(2.24239E-01-8.24608E-02*Y)*Z1
        GAS3=(-1.17615E-03-1.87566E-03*Z1-1.19155E-04*Y)*Y*Y
        GAS4=(1.18397E00+6.48520E-02*Y-5.52634E-01*Z1)*Z1*Z1
        TNON=GAS1+GAS2+GAS3+GAS4
        GO TO 240
110     IF (Z1.GT.1.4E00) GO TO 120
        GAS1=-8.12952E00-8.28637E-01*Y
        GAS2=(2.26904E01+1.41132E00*Y)*Z1
        GAS3=(-2.98633E-02+2.70066E-02*Z1-2.28103E-04*Y)*Y*Y
        GAS4=(-1.91806E01-5.78875E-01*Y+5.62580E00*Z1)*Z1*Z1
        GAS5=-3.99845E00+2.26369E-01*Y
        GAS6=(2.52876E00-7.28448E-01*Y)*Z1
        GAS7=(1.09769E-02-1.83819E-02*Z1-1.51380E-04*Y)*Y*Y
        GAS8=(2.99238E00+3.91440E-01*Y-2.04463E00*Z1)*Z1*Z1
        GAS9=EXP(-3.887015E01-2.908228E01*Y+4.070557E01*Z1
       *+2.682347E01*Y*Z1)
        GO TO 230
120     IF (Z1.GT.1.95E00) GO TO 130
        GAS1=-1.98573E01-1.67225E00*Y
        GAS2=(3.76159E01+2.10964E00*Y)*Z1
        GAS3=(-3.40174E-02+2.31712E-02*Z1-9.80275E-05*Y)*Y*Y
        GAS4=(-2.22215E01-6.44596E-01*Y+4.40486E00*Z1)*Z1*Z1
        GAS5=-5.36809E00+2.41201E-01*Y
        GAS6=(-1.25881E00-8.62744E-01*Y)*Z1
        GAS7=(-3.79774E-03-7.81335E-03*Z1-3.80005E-04*Y)*Y*Y
        GAS8=(5.58609E00+3.78963E-01*Y-1.81566E00*Z1)*Z1*Z1
        GAS9=EXP(2.08E01-2.56E01*Y+1.0E00*Z1+1.80E01*Y*Z1)
        GO TO 230
130     IF (Z1.GT.2.60E00) WRITE (6,1000) RHO,P
        GAS1=-2.33271E01-1.89958E00*Y
        GAS2=(3.21440E01+1.68622E00*Y)*Z1
        GAS3=(-4.42123E-02+2.82629E-02*Z1+6.63272E-04*Y)*Y*Y
        GAS4=(-1.38645E01-3.40976E-01*Y+2.04466E00*Z1)*Z1*Z1
        GAS5=8.35474E00+1.71347E00*Y
        GAS6=(-1.60715E01-1.63139E00*Y)*Z1
        GAS7=(4.14641E-02-2.30068E-02*Z1+1.53246E-05*Y)*Y*Y
        GAS8=(8.70275E00+3.60966E-01*Y-1.46166E00*Z1)*Z1*Z1
        GAS9=EXP(1.115884E02-6.452606E00*Y-5.337863E01*Z1
```

```
     *+2.026986E00*Y*Z1)
      GO TO 230
140   IF (Z1.GT.0.25E00) GO TO 150
      T=P/(RHO*GASCON)
      GO TO 250
150   IF (Z1.GT.0.95E00) GO TO 160
      GAS1=2.03910E-02+7.67310E-03*Y
      GAS2=(8.48581E-01-2.93086E-02*Y)*Z1
      GAS3=(8.40269E-04-1.47701E-03*Z1+3.13687E-05*Y)*Y*Y
      GAS4=(2.67251E-01+2.37262E-02*Y-1.41973E-01*Z1)*Z1*Z1
      TNON=GAS1+GAS2+GAS3+GAS4
      GO TO 240
160   IF (Z1.GT.1.45E00) GO TO 170
      GAS1=-5.12404E00-2.84740E-01*Y
      GAS2=(1.54532E01+4.52475E-01*Y)*Z1
      GAS3=(-1.22881E-02+8.56845E-03*Z1-3.25256E-04*Y)*Y*Y
      GAS4=(-1.35181E01-1.68725E-01*Y+4.18451E00*Z1)*Z1*Z1
      GAS5=7.52564E00+8.35238E-01*Y
      GAS6=(-1.95558E01-1.23393E00*Y)*Z1
      GAS7=(3.34510E-02-2.34269E-02*Z1+4.81788E-04*Y)*Y*Y
      GAS8=(1.71779E01+4.54628E-01*Y-5.09936E00*Z1)*Z1*Z1
      GAS9=EXP(6.148442E01-1.828123E01*Y-5.468755E01*Z1
     *+1.562500E01*Y*Z1)
      GO TO 230
170   IF (Z1.GT.2.05E00) GO TO 180
      GAS1=-1.23779E01-1.14728E00*Y
      GAS2=(2.41382E01+1.38957E00*Y)*Z1
      GAS3=(-3.63693E-02+2.24265E-02*Z1-3.23888E-04*Y)*Y*Y
      GAS4=(-1.42844E01-4.06553E-01*Y+2.87620E00*Z1)*Z1*Z1
      GAS5=4.40782E00+1.33046E00*Y
      GAS6=(-1.15405E01-1.59892E00*Y)*Z1
      GAS7=(5.30580E-02-3.10376E-02*Z1+4.77650E-04*Y)*Y*Y
      GAS8=(8.57309E00+4.71274E-01*Y-1.96233E00*Z1)*Z1*Z1
      GAS9=EXP(1.4075E02-6.499992E00*Y-7.75E01*Z1+5.0E00*Y*Z1)
      GO TO 230
180   IF (Z1.GT.2.50E00) WRITE (6,1000) RHO,P
      GAS1=-1.27244E01-1.66684E00*Y
      GAS2=(1.72708E01+1.45307E00*Y)*Z1
      GAS3=(-3.64515E-02+1.90463E-02*Z1+4.80787E-04*Y)*Y*Y
      GAS4=(-6.97208E00-3.04323E-01*Y+9.67524E-01*Z1)*Z1*Z1
      GAS5=7.71330E00+5.08340E-01*Y
      GAS6=(-9.82110E00-4.49138E-01*Y)*Z1
      GAS7=(-9.41787E-04-2.40293E-03*Z1-8.28450E-04*Y)*Y*Y
      GAS8=(4.16530E00+9.63923E-02*Y-5.88807E-01*Z1)*Z1*Z1
      GAS9=EXP(-1.092654E03-3.05312E02*Y+4.656243E02*Z1+
     *1.312498E02*Y*Z1)
      GO TO 230
190   IF (Z1.GT.0.25E00) GO TO 200
      T=P/(RHO*GASCON)
      GO TO 250
```

```
200   IF (Z1.GT.1.00E00) GO TO 210
      GAS1=-1.54141E-03+6.58337E-04*Y
      GAS2=(9.82201E-01-3.85028E-03*Y)*Z1
      GAS3=(1.23111E-04-4.08210E-04*Z1+2.13592E-05*Y)*Y*Y
      GAS4=(3.77441E-02+4.56963E-03*Y-2.35172E-02*Z1)*Z1*Z1
      TNON=GAS1+GAS2+GAS3+GAS4
      GO TO 240
210   IF (Z1.GT.1.45E00) GO TO 220
      GAS1=8.06492E-01+9.91293E-02*Y
      GAS2=(-1.70742E00-2.28264E-01*Y)*Z1
      GAS3=(5.03500E-03-6.13927E-03*Z1+1.69824E-04*Y)*Y*Y
      GAS4=(3.02351E00+1.31574E-01*Y-1.12755E00*Z1)*Z1*Z1
      GAS5=-1.17930E-01-2.12207E-01*Y
      GAS6=(1.36524E00+4.05886E-01*Y)*Z1
      GAS7=(-1.88260E-02+1.65486E-02*Z1-5.11400E-04*Y)*Y*Y
      GAS8=(-2.10926E00-1.89881E-01*Y+8.79806E-01*Z1)*Z1*Z1
      GAS9=EXP(1.959604E02-4.269391E01*Y-1.734931E02*Z1+
     *3.762898E01*Y*Z1)
      GO TO 230
220   IF (Z1.GT.2.3E00) WRITE (6,1000) RHO,P
      GAS1=-1.66249E00-8.91113E-02*Y
      GAS2=(4.11648E00+8.78093E-02*Y)*Z1
      GAS3=(-3.09742E-03+1.99879E-03*Z1+6.85472E-05*Y)*Y*Y
      GAS4=(-1.84445E00-7.50324E-03*Y+3.05784E-01*Z1)*Z1*Z1
      GAS5=1.11555E01+1.32100E00*Y
      GAS6=(-1.71236E01-1.29190E00*Y)*Z1
      GAS7=(6.28124E-02-3.07949E-02*Z1+1.57743E-03*Y)*Y*Y
      GAS8=(8.63804E00+3.07809E-01*Y-1.42634E00*Z1)*Z1*Z1
      GAS9=EXP(1.330611E02+8.979635E00*Y-7.265298E01*Z1
     *-2.449009E00*Y*Z1)
230   TNON=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
240   T=(10.**TNON)*TO
250   IF (IFLAG) 10,260,270
260   IF (JFLAG) 30,40,10
270   IF (JFLAG) 60,70,10
1000  FORMAT(/20X,48HWARNING!  OUTSIDE OF VALIDITY RANGE OF CURVE FIT
     *,/,20X,5HRHO =,1PE15.8,5X,3HP =,1PE15.8,/)
      END
```

```
      SUBROUTINE TGAS4 (P,RHO,H)
C
C
C     INPUTS FOR SUBROUTINE :
C
C     P = PRESSURE, IN NEWTONS/M.**2.
C     RHO = DENSITY, IN KG/M**3.
C
C     OUTPUT :
C
C     H = SPECIFIC ENTHALPY, IN (M/SEC)**2.
C
C
      DATA RO,PO/1.292E00,1.0133E05/
      Y=ALOG10(RHO/RO)
      X=ALOG10(P/PO)
      IF(ABS(Y+4.5E00).LT.2.5E-02) GO TO 20
      IF(ABS(Y+0.5E00).LT.5.0E-03) GO TO 50
      IFLAG=-1
      GO TO 90
10    RETURN
20    IFLAG=0
      RSAVE=RHO
      YM=Y
      Y=-4.5E00+2.5E-02
      YHIGH=Y
      RHO=(10.**Y)*RO
      JFLAG=-1
      GO TO 90
30    HHIGH=H
      Y=-4.5E00-2.5E-02
      YLOW=Y
      RHO=(10.**Y)*RO
      JFLAG=0
      GO TO 90
40    HLOW=H
      GO TO 80
50    IFLAG=1
      RSAVE=RHO
      YM=Y
      Y=-0.5E00+0.5E-02
      YHIGH=Y
      RHO=(10.**Y)*RO
      JFLAG=-1
      GO TO 90
60    HHIGH=H
      Y=-0.5E00-0.5E-02
      YLOW=Y
      RHO=(10.**Y)*RO
      JFLAG=0
```

```
      GO TO 90
70    HLOW=H
80    H=HLOW+(HHIGH-HLOW)/(YHIGH-YLOW)*(YM-YLOW)
      RHO=RSAVE
      GO TO 10
90    Z1=X-Y
      IF (Y.GT.-0.5E00) GO TO 190
      IF (Y.GT.-4.5E00) GO TO 140
      IF (Z1.GT.0.10E00) GO TO 100
      GAMM=1.3986E00
      GO TO 240
100   IF (Z1.GT.0.85E00) GO TO 110
      GAS1=2.53908E02+1.01491E02*Y
      GAS2=(-3.87199E02-1.54304E02*Y)*Z1
      GAS3=(7.28532E00-8.04378E00*Z1-1.82577E-03*Y)*Y*Y
      GAS4=(9.86233E01+4.63763E01*Y+2.18994E01*Z1)*Z1*Z1
      GAS5=-2.52423E02-1.01445E02*Y
      GAS6=(3.87210E02+1.54298E02*Y)*Z1
      GAS7=(-7.2773E00+8.04277E00*Z1+2.28399E-03*Y)*Y*Y
      GAS8=(-9.87576E01-4.63883E01*Y-2.19438E01*Z1)*Z1*Z1
      GAS9=EXP(-11.E00+2.E00*Y+11.E00*Z1-2.E00*Y*Z1)
      GAMM=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.-GAS9)
      GO TO 240
110   IF (Z1.GT.1.30E00) GO TO 120
      GAS1=-1.05745E01-1.93693E00*Y
      GAS2=(3.07202E01+3.35578E00*Y)*Z1
      GAS3=(-7.79965E-02+6.68790E-02*Z1-9.86882E-04*Y)*Y*Y
      GAS4=(-2.60637E01-1.42391E00*Y+7.23223E00*Z1)*Z1*Z1
      GAS5=-1.86342E01+2.41997E-02*Y
      GAS6=(3.20880E01-7.46914E-01*Y)*Z1
      GAS7=(3.75161E-02-4.10125E-02*Z1+5.74637E-04*Y)*Y*Y
      GAS8=(-1.69985E01+5.39041E-01*Y+2.56253E00*Z1)*Z1*Z1
      GAS9=EXP(2.768567E02+2.152383E01*Y-2.164837E02*Z1
     *-1.394837E01*Y*Z1)
      GO TO 230
120   IF (Z1.GT.1.95E00) GO TO 130
      GAS1=6.17584E-01-2.40690E-01*Y
      GAS2=(1.95904E00+3.41644E-01*Y)*Z1
      GAS3=(-1.01073E-02+6.77631E-03*Z1-1.15922E-04*Y)*Y*Y
      GAS4=(-1.68951E00-1.10932E-01*Y+4.26058E-01*Z1)*Z1*Z1
      GAS5=-1.34222E01-5.43713E-01*Y
      GAS6=(1.81528E01+3.95928E-01*Y)*Z1
      GAS7=(-7.41105E-03+1.67768E-03*Z1-3.32714E-06*Y)*Y*Y
      GAS8=(-7.97425E00-5.80593E-02*Y+1.12448E00*Z1)*Z1*Z1
      GAS9=EXP(8.677803E01-8.370349E00*Y-4.074084E01*Z1
     *+7.407405E00*Y*Z1)
      GO TO 230
130   IF (Z1.GT.2.60E00) WRITE (6,1000) RHO,P
      GAS1=-8.32595E00-3.50219E-01*Y
      GAS2=(1.36455E01+3.59350E-01*Y)*Z1
```

```
      GAS3=(-3.70109E-03+3.30836E-03*Z1+1.10018E-04*Y)*Y*Y
      GAS4=(-6.49007E00-8.38594E-02*Y+1.02443E00*Z1)*Z1*Z1
      GAS5=-3.08441E01-1.49510E00*Y
      GAS6=(3.00585E01+9.19650E-01*Y)*Z1
      GAS7=(-3.60024E-02+1.02522E-02*Z1-4.68760E-04*Y)*Y*Y
      GAS8=(-9.33522E00-1.35228E-01*Y+8.92634E-01*Z1)*Z1*Z1
      GAS9=EXP(8.800047E01-1.679356E01*Y-3.333353E01*Z1
     *+8.465574E00*Y*Z1)
      GO TO 230
140   IF (Z1.GT.0.1E00) GO TO 150
      GAMM=1.399E00
      GO TO 240
150   IF (Z1.GT.0.95E00) GO TO 160
      GAS1=-1.33083E02-9.98707E00*Y
      GAS2=(3.94734E02+2.35810E01*Y)*Z1
      GAS3=(1.43957E00-1.43175E00*Z1+1.77068E-05*Y)*Y*Y
      GAS4=(-3.84712E02-1.36367E01*Y+1.24325E02*Z1)*Z1*Z1
      GAS5=1.34486E02+9.99122E00*Y
      GAS6=(-3.94719E02-2.35853E01*Y)*Z1
      GAS7=(-1.43799E00+1.43039E00*Z1+1.44367E-04*Y)*Y*Y
      GAS8=(3.84616E02+1.36318E01*Y-1.24348E02*Z1)*Z1*Z1
      GAS9=EXP(-2.141444E01+1.381584E00*Y+2.039473E01*Z1
     *-1.315789E00*Y*Z1)
      GAMM=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.-GAS9)
      GO TO 240
160   IF (Z1.GT.1.50E00) GO TO 170
      GAS1=-7.36684E00-1.13247E00*Y
      GAS2=(2.47879E01+1.99625E00*Y)*Z1
      GAS3=(-4.91630E-02+4.16673E-02*Z1-6.58149E-04*Y)*Y*Y
      GAS4=(-2.32990E01-8.59418E-01*Y+7.19016E00*Z1)*Z1*Z1
      GAS5=-2.42647E00+5.57912E-01*Y
      GAS6=(-2.03055E00-1.22031E00*Y)*Z1
      GAS7=(3.74866E-02-3.39278E-02*Z1+5.21042E-04*Y)*Y*Y
      GAS8=(7.75414E00+6.08488E-01*Y-3.68326E00*Z1)*Z1*Z1
      GAS9=EXP(8.077385E01-1.273807E01*Y-6.547623E01*Z1
     *+1.190475E01*Y*Z1)
      GO TO 230
170   IF (Z1.GT.2.00E00) GO TO 180
      GAS1=4.31520E-01-2.83857E-01*Y
      GAS2=(2.27791E00+3.99159E-01*Y)*Z1
      GAS3=(-1.29444E-02+8.78724E-03*Z1-1.60583E-04*Y)*Y*Y
      GAS4=(-1.84314E00-1.28136E-01*Y+4.45362E-01*Z1)*Z1*Z1
      GAS5=-1.03883E01-3.58718E-01*Y
      GAS6=(1.35068E01+1.87268E-01*Y)*Z1
      GAS7=(-4.28184E-03-9.52016E-04*Z1-4.10506E-05*Y)*Y*Y
      GAS8=(-5.63894E00-1.45626E-03*Y+7.39915E-01*Z1)*Z1*Z1
      GAS9=EXP(2.949221E02+1.368660E01*Y-1.559335E02*Z1
     *-3.787766E00*Y*Z1)
      GO TO 230
180   IF (Z1.GT.2.5E00) WRITE (6,1000) RHO,P
```

```
      GAS1=-3.77766E00-5.53738E-01*Y
      GAS2=(6.60834E00+4.87181E-01*Y)*Z1
      GAS3=(-2.11045E-02+9.67277E-03*Z1-2.19420E-04*Y)*Y*Y
      GAS4=(-2.94754E00-1.02365E-01*Y+4.39620E-01*Z1)*Z1*Z1
      GAS5=4.05813E01+3.25692E00*Y
      GAS6=(-4.79583E01-2.53660E00*Y)*Z1
      GAS7=(9.06436E-02-3.47578E-02*Z1+1.00077E-03*Y)*Y*Y
      GAS8=(1.89040E01+4.94114E-01*Y-2.48554E00*Z1)*Z1*Z1
      GAS9=EXP(5.34718E02+7.495657E01*Y-2.219822E02*Z1
     *-3.017229E01*Y*Z1)
      GO TO 230
190   IF (Z1.GT.0.1E00) GO TO 200
      GAMM=1.4017E00
      GO TO 240
200   IF (Z1.GT.1.05E00) GO TO 210
      GAS1=-9.67488E01+2.05296E-01*Y
      GAS2=(2.69927E02-1.92887E00*Y)*Z1
      GAS3=(3.78392E-01-3.24965E-01*Z1-3.61036E-03*Y)*Y*Y
      GAS4=(-2.46711E02+1.54416E00*Y+7.48760E01*Z1)*Z1*Z1
      GAS5=9.81502E01-2.05448E-01*Y
      GAS6=(-2.69913E02+1.93052E00*Y)*Z1
      GAS7=(-3.78527E-01+3.24832E-01*Z1+3.66182E-03*Y)*Y*Y
      GAS8=(2.46630E02-1.54646E00*Y-7.48980E01*Z1)*Z1*Z1
      GAS9=EXP(-2.659865E01+1.564631E00*Y+2.312926E01*Z1
     *-1.360543E00*Y*Z1)
      GAMM=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.-GAS9)
      GO TO 240
210   IF (Z1.GT.1.60E00) GO TO 220
      GAS1=-2.67593E-01-1.87457E-01*Y
      GAS2=(5.07693E00+2.72286E-01*Y)*Z1
      GAS3=(1.04541E-02-1.42211E-02*Z1+6.38962E-04*Y)*Y*Y
      GAS4=(-5.08520E00-7.81935E-02*Y+1.58711E00*Z1)*Z1*Z1
      GAS5=2.87969E00+3.9009E-01*Y
      GAS6=(-8.06179E00-5.51250E-01*Y)*Z1
      GAS7=(-1.01903E-02+1.35906E-02*Z1-8.97772E-04*Y)*Y*Y
      GAS8=(7.29592E00+1.83861E-01*Y-2.15153E00*Z1)*Z1*Z1
      GAS9=EXP(1.828573E02-3.428596E01*Y-1.51786E02*Z1
     *+2.976212E01*Y*Z1)
      GO TO 230
220   IF (Z1.GT.2.30E00) WRITE (6,1000) RHO,P
      GAS1=9.21537E-01-2.39670E-01*Y
      GAS2=(1.30714E00+3.42990E-01*Y)*Z1
      GAS3=(-2.18847E-02+1.36691E-02*Z1-4.90274E-04*Y)*Y*Y
      GAS4=(-1.20916E00-1.10206E-01*Y+3.087920E-01*Z1)*Z1*Z1
      GAS5=-6.77089E00-6.90476E-02*Y
      GAS6=(8.18168E00-9.52708E-02*Y)*Z1
      GAS7=(2.98487E-02-1.78706E-02*Z1+6.28419E-04*Y)*Y*Y
      GAS8=(-3.07662E00+6.60408E-02*Y+3.38590E-01*Z1)*Z1*Z1
      GAS9=EXP(1.5916669E02+3.976192E01*Y-7.966199E01*Z1
     *-1.66667E01*Y*Z1)
```

```
230    GAMM=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
240    H=GAMM/(GAMM-1.0E00)*P/RHO
       IF (IFLAG) 10,250,260
250    IF (JFLAG) 30,40,10
260    IF (JFLAG) 60,70,10
1000   FORMAT(/20X,48HWARNING!  OUTSIDE OF VALIDITY RANGE OF CURVE FIT
      *,/,20X,5HRHO =,1PE15.8,5X,3HP =,1PE15.8,/)
       END
```

```
      SUBROUTINE TGAS5(P,S,RHO)
C
C
C     INPUTS FOR SUBROUTINE:
C     P=PRESSURE IN NEWTONS/M**2.
C     S=SPECIFIC ENTROPY IN (M/SEC)**2./KELVIN
C
C     OUTPUT:
C     RHO=DENSITY IN KG/M**3.
C
C
      DATA P0,R0,S0,GASCON/1.0133E05,1.292E00,6779.2E00,287.06E00/
      X=ALOG10(P/P0)
      SNON=ALOG10(S/GASCON)
      Z=X-SNON
      IF (SNON.GE.1.23E00) GO TO 10
      DELTS=S-S0
      RRATIO=ALOG(P/P0)/1.4-DELTS/(3.5*GASCON)
      RHO=R0*EXP(RRATIO)
      RETURN
10    IF (SNON.GE.1.42E00) GO TO 20
      GAS1=-1.72119E01+5.49354E01*SNON
      GAS2=(-1.99776E00+3.17884E00*SNON)*Z
      GAS3=(-4.69831E01-8.66580E-01*Z+1.21069E01*SNON)*SNON*SNON
      GAS4=(1.58567E-01-1.03055E-01*SNON-1.52322E-03*Z)*Z*Z
      Y=GAS1+GAS2+GAS3+GAS4
      GO TO 120
20    IF (SNON.GE.1.592E00) GO TO 30
      GAS1=-2.78074E02+6.11791E02*SNON
      GAS2=(1.37528E01-1.92394E01*SNON)*Z
      GAS3=(-4.42909E02+7.10425E00*Z+1.05869E02*SNON)*SNON*SNON
      GAS4=(-1.97269E-01+1.49708E-01*SNON+1.19153E-03*Z)*Z*Z
      GAS5=2.80393E02-5.95834E02*SNON
      GAS6=(-1.75934E01+2.49706E01*SNON)*Z
      GAS7=(4.21767E02-8.85461E00*Z-9.94515E01*SNON)*SNON*SNON
      GAS8=(3.61896E-01-2.55458E-01*SNON-2.96892E-03*Z)*Z*Z
      GAS9=EXP(-15.0*(X+54.179-86.947*SNON+33.583*SNON*SNON))
      GO TO 110
30    IF (SNON.GE.1.70E00) GO TO 50
      ZM=7.5269E00*SNON-14.9366
      IF (Z.GE.ZM) GO TO 40
      GAS1=1.10732E02-1.33968E02*SNON
      GAS2=(3.7583E-01+2.77887E-01*SNON)*Z
      GAS3=4.03018E01*SNON*SNON
      GAS4=(1.18506E-01-6.98812E-02*SNON)*Z*Z
      GAS5=-6.85292E01+8.23834E01*SNON
      GAS6=(-1.24942E00+8.31615E-01*SNON)*Z
      GAS7=-2.4524E01*SNON*SNON
      GAS8=(-1.13019E-01+7.46719E-02*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+33.976*SNON-49.659))
```

```
      GO TO 110
40    GAS1=3.97149E01-4.80988E01*SNON
      GAS2=(-1.17821E00+1.27522E00*SNON)*Z
      GAS3=1.45552E01*SNON*SNON
      GAS4=(-2.50279E-01+1.58399E-01*SNON)*Z*Z
      GAS5=-5.01859E01+6.31564E01*SNON
      GAS6=(2.39925E00-1.47883E00*SNON)*Z
      GAS7=-1.98852E01*SNON*SNON
      GAS8=(4.66791E-01-3.06926E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+11.16*SNON-18.203))
      GO TO 110
50    IF (SNON.GE.1.80E00) GO TO 70
      ZM=7.5269E00*SNON-14.9366
      IF (Z.GE.ZM) GO TO 60
      GAS1=6.22639E01-7.8884E01*SNON
      GAS2=(-2.19123E00+1.84533E00*SNON)*Z
      GAS3=2.48413E01*SNON*SNON
      GAS4=(-8.76105E-02+5.55047E-02*SNON)*Z*Z
      GAS5=-7.28463E01+9.42375E01*SNON
      GAS6=(4.04979E00-2.45467E00*SNON)*Z
      GAS7=-3.02865E01*SNON*SNON
      GAS8=(1.2872E-01-8.63721E-02*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+13.4576*SNON-18.36))
      GO TO 110
60    GAS1=-2.99578E01+3.89998E01*SNON
      GAS2=(2.90256E00-1.25088E00*SNON)*Z
      GAS3=-1.26641E01*SNON*SNON
      GAS4=(6.13867E-02-5.17059E-02*SNON)*Z*Z
      GAS5=6.05677E01-7.08307E01*SNON
      GAS6=(-2.95622E00+1.70504E00*SNON)*Z
      GAS7=2.07549E01*SNON*SNON
      GAS8=(-2.70962E-01+1.80057E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+16.822*SNON-29.139))
      GO TO 110
70    IF (SNON.GE.1.90E00) GO TO 80
      GAS1=6.23124E02-6.77571E02*SNON
      GAS2=(9.12811E01-9.55480E01*SNON)*Z
      GAS3=(1.84603E02+2.54274E01*Z)*SNON*SNON
      GAS4=(6.97635E-01-3.27916E-01*SNON+4.90838E-03*Z)*Z*Z
      GAS5=-6.39514E02+6.97458E02*SNON
      GAS6=(-1.00154E02+1.06701E02*SNON)*Z
      GAS7=(-1.90745E02-2.86323E01*Z)*SNON*SNON
      GAS8=(-4.81471E-01+2.00348E-01*SNON-6.43371E-03*Z)*Z*Z
      GAS9=EXP(-2.0*(Z+35.275*SNON-58.624))
      GO TO 110
80    IF (SNON.GE.2.00E00) GO TO 90
      GAS1=1.40088E01-1.58855E01*SNON
      GAS2=(1.71245E-01+4.32352E-01*SNON)*Z
      GAS3=4.42836E00*SNON*SNON
      GAS4=(-9.54417E-03+8.92335E-03*SNON)*Z*Z
```

```
      GAS5=-1.13217E02+1.24304E02*SNON
      GAS6=(4.41505E00-2.48530E00*SNON)*Z
      GAS7=-3.42370E01*SNON*SNON
      GAS8=(-7.84297E-02+1.21459E-02*SNON)*Z*Z
      GAS9=EXP(-2.00*(Z+20.884*SNON-36.54))
      GO TO 110
90    IF (SNON.GT.2.10) GO TO 100
      GAS1=-3.62767E01+3.8634E01*SNON
      GAS2=(1.48507E00-3.41824E-01*SNON)*Z
      GAS3=-1.04718E01*SNON*SNON
      GAS4=(-3.25437E-02+1.22032E-02*SNON)*Z*Z
      GAS5=(6.52920E01-6.28154E01*SNON)
      GAS6=(2.10906E00-1.11759E00*SNON)*Z
      GAS7=1.50026E01*SNON*SNON
      GAS8=(2.71716E-01-1.4431E-01*SNON)*Z*Z
      GAS9=EXP(-5.00*(Z+28.284*SNON-53.185))
      GO TO 110
100   GAS1=1.86938E02-1.45261E02*SNON
      GAS2=(2.23883E01-1.04974E01*SNON)*Z
      GAS3=2.60284E01*SNON*SNON
      GAS4=(1.85949E00-8.98218E-01*SNON)*Z*Z
      GAS5=-1.27867E02+8.58453E01*SNON
      GAS6=(-2.28808E01+1.13034E01*SNON)*Z
      GAS7=-1.1234E01*SNON*SNON
      GAS8=(-1.56005E00+7.79397E-01*SNON)*Z*Z
      GAS9=EXP(-2.00*(Z+15.048*SNON-26.307))
110   Y=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
120   RHO=(10.**Y)*R0
      RETURN
      END
```

```
      SUBROUTINE TGAS6(P,S,E)
C
C     INPUTS FOR SUBROUTINE:
C     P=PRESSURE IN NEWTONS/M**2.
C     S=ENTROPY IN (M/SEC)**2./KELVIN
C
C     OUTPUT:
C     E=INTERNAL ENERGY IN (M/SEC)**2.
C
C
      DATA P0,E0,S0,GASCON/1.0133E05,78408.4E00,6779.2E00,287.06E00/
      X=ALOG10(P/P0)
      SNON=ALOG10(S/GASCON)
      Z=X-SNON
      IF (SNON.GE.1.23E00) GO TO 10
      DELTS=S-S0
      ERATIO=(ALOG(P/P0)+DELTS/GASCON)/3.5E00
      E=2.5E00*E0*EXP(ERATIO)
      RETURN
 10   IF (SNON.GT.1.4E00) GO TO 20
      GAS1=4.89511E01-1.15989E02*SNON
      GAS2=(-4.43026E-01+1.49819E00*SNON)*Z
      GAS3=(8.70709E01-7.02417E-01*Z-2.04916E01*SNON)*SNON*SNON
      GAS4=(-5.09747E-02+2.94929E-02*SNON+5.24685E-04*Z)*Z*Z
      Y=GAS1+GAS2+GAS3+GAS4
      GO TO 120
 20   IF (SNON.GE.1.592E00) GO TO 30
      GAS1=1.30563E02-2.87254E02*SNON
      GAS2=(-7.71873E00+1.17605E01*SNON)*Z
      GAS3=(2.06757E02-4.31838E00*Z-4.83472E01*SNON)*SNON*SNON
      GAS4=(1.64299E-01-1.21768E-01*SNON-1.26118E-03*Z)*Z*Z
      GAS5=-8.4598E01+1.74701E02*SNON
      GAS6=(9.52234E00-1.32819E01*SNON)*Z
      GAS7=(-1.20234E02+4.6271E00*Z+2.75818E01*SNON)*SNON*SNON
      GAS8=(-2.27725E-01+1.60365E-01*SNON+1.4933E-03*Z)*Z*Z
      GAS9=EXP(-15.0*(X+54.179-86.947*SNON+33.583*SNON*SNON))
      GO TO 110
 30   IF (SNON.GE.1.70E00) GO TO 50
      ZM=-1.917E00*SNON+0.092E00
      IF (Z.GT.ZM) GO TO 40
      GAS1=-1.34875E02+1.92654E02*SNON
      GAS2=(1.04478E01-6.60355E00*SNON)*Z
      GAS3=-6.72013E01*SNON*SNON
      GAS4=(4.14546E-01-2.75698E-01*SNON)*Z*Z
      GAS5=1.07768E02-1.61662E02*SNON
      GAS6=(-1.01257E01+6.4962E00*SNON)*Z
      GAS7=5.93194E01*SNON*SNON
      GAS8=(-4.63204E-01+3.04448E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+50.04*SNON-74.698))
      GO TO 110
```

```
40    GAS1=-3.39933E01+4.02792E01*SNON
      GAS2=(7.62742E-01-3.79044E-01*SNON)*Z
      GAS3=-1.09895E01*SNON*SNON
      GAS4=(-7.13412E-02+4.22359E-02*SNON)*Z*Z
      GAS5=3.32182E01-4.03847E01*SNON
      GAS6=(1.87601E-01-1.07599E-01*SNON)*Z
      GAS7=1.22657E01*SNON*SNON
      GAS8=(-1.65537E-01+1.02560E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+14.062*SNON-24.046))
      GO TO 110
50    IF (SNON.GE.1.80E00) GO TO 70
      ZM=-1.917E00*SNON+0.092E00
      IF (Z.GT.ZM) GO TO 60
      GAS1=-1.66572E01+1.51243E01*SNON
      GAS2=(-4.72897E-01+2.95848E-01*SNON)*Z
      GAS3=-2.31418E00*SNON*SNON
      GAS4=(-3.61191E-02+1.7288E-02*SNON)*Z*Z
      GAS5=-3.27168E00+9.45383E00*SNON
      GAS6=(2.04E00-1.1342E00*SNON)*Z
      GAS7=-4.28765E00*SNON*SNON
      GAS8=(1.5021E-01-8.27388E-02*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+16.995*SNON-23.354))
      GO TO 110
60    GAS1=-1.94882E01+2.31472E01*SNON
      GAS2=(1.00783E00-5.18103E-01*SNON)*Z
      GAS3=-5.92829E00*SNON*SNON
      GAS4=(9.92242E-02-5.67928E-02*SNON)*Z*Z
      GAS5=-2.09813E00+1.22781E00*SNON
      GAS6=(-6.27531E-01+3.71651E-01*SNON)*Z
      GAS7=9.84125E-04*SNON*SNON
      GAS8=(-1.24024E-01+7.65034E-02*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+18.152*SNON-31.096))
      GO TO 110
70    IF (SNON.GE.1.90E00) GO TO 80
      GAS1=-6.37715E01+5.46005E01*SNON
      GAS2=(-8.90073E00+5.02063E00*SNON)*Z
      GAS3=-9.67094E00*SNON*SNON
      GAS4=(-7.13493E-01+3.97962E-01*SNON)*Z*Z
      GAS5=5.05131E01-3.97363E01*SNON
      GAS6=(9.27248E00-5.15576E00*SNON)*Z
      GAS7=6.45632E00*SNON*SNON
      GAS8=(7.2563E-01-4.03054E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+35.275*SNON-58.624))
      GO TO 110
80    IF (SNON.GE.2.00E00) GO TO 90
      GAS1=-1.75434E01+2.00307E01*SNON
      GAS2=(6.20484E-01-2.76756E-01*SNON)*Z
      GAS3=-4.76084E00*SNON*SNON
      GAS4=(1.14921E-02-5.80043E-03*SNON)*Z*Z
      GAS5=1.10485E01-1.261E01*SNON
```

```
      GAS6=(-6.59043E-01+3.66266E-01*SNON)*Z
      GAS7=3.59417E00*SNON*SNON
      GAS8=(-6.04061E-03+6.78382E-03*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+20.884*SNON-36.54))
      GO TO 110
90    IF (SNON.GE.2.10E00) GO TO 100
      GAS1=-1.18487E01+1.26558E01*SNON
      GAS2=(6.09885E-04+5.90396E-02*SNON)*Z
      GAS3=-2.45275E00*SNON*SNON
      GAS4=(-6.70888E-03+5.21033E-03*SNON)*Z*Z
      GAS5=3.39157E00-3.835E00*SNON
      GAS6=(-2.14031E-01+9.90508E-02*SNON)*Z
      GAS7=1.07008E00*SNON*SNON
      GAS8=(-7.87516E-02+3.63642E-02*SNON)*Z*Z
      GAS9=EXP(-5.0*(Z+28.284*SNON-53.185))
      GO TO 110
100   GAS1=-2.40656E01+3.35589E00*SNON
      GAS2=(-1.14521E01+5.62572E00*SNON)*Z
      GAS3=4.93842E00*SNON*SNON
      GAS4=(-7.55608E-01+3.69774E-01*SNON)*Z*Z
      GAS5=2.23905E01-1.90842E00*SNON
      GAS6=(1.05618E01-5.17247E00*SNON)*Z
      GAS7=-4.3714E00*SNON*SNON
      GAS8=(6.27818E-01-3.12015E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+15.048*SNON-26.307))
110   Y=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
120   E=(10.**Y)*E0
      RETURN
      END
```

```
      SUBROUTINE TGAS7(P,S,A)
C
C     INPUTS FOR SUBROUTINE:
C     P=PRESSURE IN NEWTONS/M**2.
C     S=ENTROPY IN (M/SEC)**2./KELVIN
C
C     OUTPUT:
C     A=SPEED OF SOUND IN M/SEC.
C
C
      DATA P0,S0,R0,A0,GASCON/1.0133E05,6779.2E00,1.292E00,331.3613E00,
     *287.06E00/
      X=ALOG10(P/P0)
      SNON=ALOG10(S/GASCON)
      Z=X-SNON
      IF (SNON.GE.1.23E00) GO TO 10
      DELTS=S-S0
      ASQLOG=ALOG(1.4E00*P0/R0)+(ALOG(P/P0)+DELTS/GASCON)/3.5E00
      A=EXP(ASQLOG/2.0E00)
      RETURN
10    IF (SNON.GT.1.4E00) GO TO 20
      GAS1=-1.38377E-01-8.84138E00*SNON
      GAS2=(2.61050E00-3.16535E00*SNON)*Z
      GAS3=(1.10866E01+9.88389E-01*Z-3.25761E00*SNON)*SNON*SNON
      GAS4=(-1.00224E-01+6.62193E-02*SNON+8.20610E-04*Z)*Z*Z
      Y=GAS1+GAS2+GAS3+GAS4
      GO TO 120
20    IF (SNON.GE.1.595E00) GO TO 30
      GAS1=1.31057E02-2.88847E02*SNON
      GAS2=(-5.04887E00+7.73862E00*SNON)*Z
      GAS3=(2.10147E02-2.88963E00*Z-5.0396E01*SNON)*SNON*SNON
      GAS4=(5.48031E-02-4.39459E-02*SNON-2.10202E-04*Z)*Z*Z
      GAS5=-1.33465E02+2.84739E02*SNON
      GAS6=(7.57389E00-1.07749E01*SNON)*Z
      GAS7=(-2.02362E02+3.8313E00*Z+4.79075E01*SNON)*SNON*SNON
      GAS8=(-1.53453E-01+1.08531E-01*SNON+9.7931E-04*Z)*Z*Z
      GAS9=EXP(-15.0*(X+54.179-86.947*SNON+33.583*SNON*SNON))
      GO TO 110
30    IF (SNON.GE.1.693E00) GO TO 50
      ZM=-9.842E00*SNON+14.19E00
      IF (Z.GT.ZM) GO TO 40
      GAS1=-6.13548E01+7.80742E01*SNON
      GAS2=(2.08524E00-1.21609E00*SNON)*Z
      GAS3=-2.43686E01*SNON*SNON
      GAS4=(8.77563E-02-5.46311E-02*SNON)*Z*Z
      GAS5=2.07952E01-2.71591E01*SNON
      GAS6=(-7.43673E-01+4.08312E-01*SNON)*Z
      GAS7=8.68124E00*SNON*SNON
      GAS8=(-9.3592E-02+5.32328E-02*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+38.785E00*SNON-57.157E00))
```

```
      GO TO 110
40    GAS1=3.37056E00-4.87016E00*SNON
      GAS2=(-3.85754E-01+2.87192E-01*SNON)*Z
      GAS3=2.02041E00*SNON*SNON
      GAS4=(-4.63144E-03+8.30832E-03*SNON)*Z*Z
      Y=GAS1+GAS2+GAS3+GAS4
      GO TO 120
50    IF (SNON.GE.1.80) GO TO 70
      ZM=-1.917*SNON+0.092
      IF (Z.GT.ZM) GO TO 60
      GAS1=-8.04927E01+7.63739E01*SNON
      GAS2=(-9.381E00+5.72104E00*SNON)*Z
      GAS3=-1.63435E01*SNON*SNON
      GAS4=(-7.48578E-01+4.50043E-01*SNON)*Z*Z
      GAS5=8.34054E01-8.58837E01*SNON
      GAS6=(4.84197E00-3.11188E00*SNON)*Z
      GAS7=2.11196E01*SNON*SNON
      GAS8=(2.33945E-01-1.59099E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+7.874*SNON-7.569))
      GO TO 110
60    GAS1=-4.73308E00+4.69363E00*SNON
      GAS2=(9.43798E-02+3.54953E-03*SNON)*Z
      GAS3=-7.98293E-01*SNON*SNON
      GAS4=(2.02561E-02-8.73036E-03*SNON)*Z*Z
      Y=GAS1+GAS2+GAS3+GAS4
      GO TO 120
70    IF (SNON.GE.1.900E00) GO TO 80
      GAS1=-6.60574E02+7.38042E02*SNON
      GAS2=(-8.77589E01+9.7894E01*SNON)*Z
      GAS3=(-2.06156E02-2.73753E01*Z)*SNON*SNON
      GAS4=-2.14028E-02*Z*Z
      GAS5=6.65014E02-7.43416E02*SNON
      GAS6=(8.87679E01-9.90508E01*SNON)*Z
      GAS7=(2.08117E02+2.77374E01*Z)*SNON*SNON
      GAS8=2.81148E-02*Z*Z
      GAS9=EXP(-2.0*(Z+35.275*SNON-58.624))
      GO TO 110
80    IF (SNON.GE.2.00E00) GO TO 90
      GAS1=-5.93554E00-7.79929E00*SNON
      GAS2=(-7.23618E00+3.31162E00*SNON)*Z
      GAS3=5.06381E00*SNON*SNON
      GAS4=(-5.377735E-01+2.46865E-01*SNON)*Z*Z
      GAS5=3.95260E01-2.90994E01*SNON
      GAS6=(6.24136E00-2.70007E00*SNON)*Z
      GAS7=5.42786E00*SNON*SNON
      GAS8=(5.95235E-01-2.67771E-01*SNON)*Z*Z
      GAS9=EXP(-2.0*(Z+13.6751*SNON-20.1676))
      GO TO 110
90    IF (SNON.GE.2.10E00) GO TO 100
      GAS1=7.02453E01-7.34732E01*SNON
```


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      GAS2=-5.86844E-01*Z
      GAS3=(1.95548E01+1.81521E-01*Z)*SNON*SNON
      GAS4=7.36786E-03*Z*Z
      GAS5=-1.69610E01+1.80169E01*SNON
      GAS6=-3.65779E-01*Z
      GAS7=(-4.75063E00+1.07583E-01*Z)*SNON*SNON
      GAS8=2.01803E-02*Z*Z
      GAS9=EXP(-5.0*(Z+28.284*SNON-53.185))
      GO TO 110
100   GAS1=-1.12793E02+1.35296E02*SNON
      GAS2=2.74295E00*Z
      GAS3=(-4.08171E01-1.29257E00*Z)*SNON*SNON
      GAS4=-2.25652E-01*Z*Z
      GAS5=2.57057E02-2.83199E02*SNON
      GAS6=-5.84656E00*Z
      GAS7=(7.91574E01+2.17312E00*Z)*SNON*SNON
      GAS8=3.35658E-01*Z*Z
      GAS9=EXP(-2.0*(Z+15.048*SNON-26.307))
110   Y=GAS1+GAS2+GAS3+GAS4+(GAS5+GAS6+GAS7+GAS8)/(1.+GAS9)
120   A=(10.**Y)*A0
      RETURN
      END
```